

Far-field approximation in two-dimensional slab-waveguides

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ABSTRACT

In this paper, we investigate the criteria for far-field approximation in a 2D problem, including the phase criterion. Using a silicon-lab as the platform, we will also compare these criteria with those of a 3D scattering problem for both sub-wavelength and large objects. The convergence of the exact solution, based on Hankel functions, and the far-field approximation are presented.

Keywords: on-chip field propagation, far-field approximation, slab waveguide

1. INTRODUCTION

In several integrated optical devices such as optical switches [1] and AWGs, an input optical beam is split into several arms and properly phase shifted. The output waveguides of the OPA are connected to a slab waveguide in which the field propagate freely in two-dimensions and is confined in the direction perpendicular to the array's plane. These setups may be a first step towards optical phased arrays in free-space, when properly combined with optical antennas, in order to control the near and far-field radiation on-chip.

In this paper, we model field propagation generated by an array of point sources confined in a silicon slab waveguide. The silicon slab thickness is chosen to be small enough for single mode operation. We derive far field conditions for such 2D radiation, in order to use simple array factors to determine the on-chip radiation from OPAs. Finally, we fabricate an on-chip 3-element OPA and present an experimental setup to observe the 2D far field pattern. Using a 1x3 Multimode Interference (MMI) coupler, we feed the array elements with uniform power while creating a phase shift in the array. Then we compare the observed far field interference pattern with the theoretical calculations. We show that the observation plane is in fact in the far field zone. Therefore, this is the first observation of a 2D optical far-field interference pattern.

Although we investigate 2D propagation in a silicon nanomembrane slab waveguide, the results of this study can be used for problems that involve free propagation in 2D and confinement in the third direction such as [2], in which a surface plasmon polariton (SPP) wave propagation at the surface of a metallic film is studied. The presented work here may be interpreted as an application of the theory of Flatland Optics [3].

2. TWO-DIMENSIONAL FAR-FIELD FORMULATION

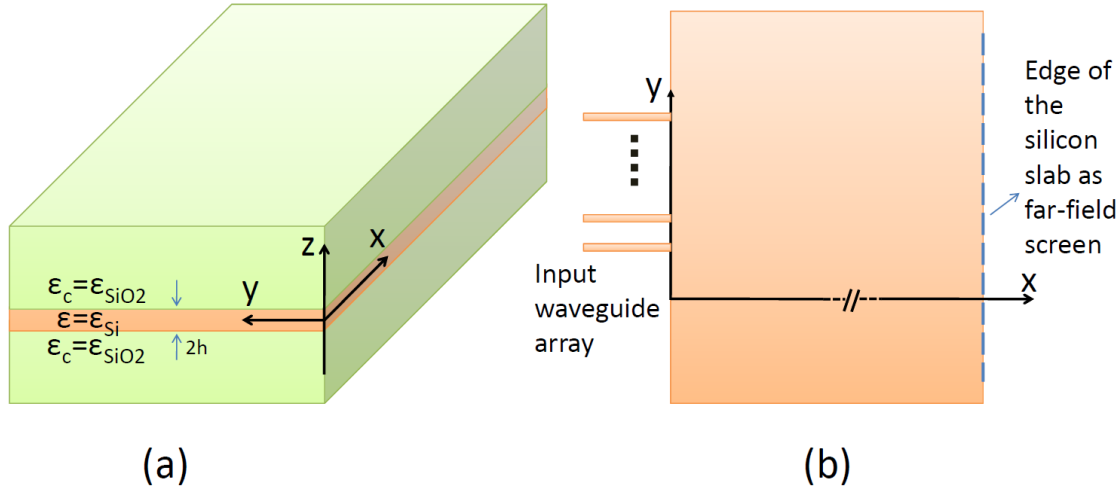


Fig 1. Geometry of the silicon slab waveguide. (b) a schematic of the waveguide array connected to a large slab waveguide.

In this section, we model the field radiated by a point source or an array of point sources in a slab dielectric waveguide. The vertically symmetric structure shown in Fig. 1 is well-known to support the Transverse Electric (TE, $E_x=E_z=H_y=0$) mode without a cut-off thickness. We assume that the slab thickness is small enough to allow only single mode operation in the z direction. We focus here on TE-polarized propagation, assuming that a point source excites this slab waveguide with field distribution: $E_y = f(z)\delta(x)\delta(y)$, where $f(z)$ is the field distribution along z , given as

$$f(z) = \begin{cases} A \cos(\beta_z h) \exp[-\alpha(z-h)] & z > h \\ A \cos(\beta_z z) & |z| \leq h \\ A \cos(\beta_z h) \exp[\alpha(z+h)] & z < -h \end{cases}$$

with well-known dispersion relation:

$(h\sqrt{\epsilon\mu\omega^2 - \beta_\rho^2}) \tan(h\sqrt{\epsilon\mu\omega^2 - \beta_\rho^2}) = -ih\sqrt{\epsilon_c\mu\omega^2 - \beta_\rho^2}$, where $\beta_\rho^2 = \beta_x^2 + \beta_y^2$, $\alpha = -i(\epsilon_c\mu\omega^2 - \beta_\rho^2)^{1/2}$. In order to calculate the field emitted by such a point source in the slab, we notice that a TE ($E_x=E_z=H_y=0$) polarized line source along the y -axis in the form of $E_y = f(z) \exp(-j\beta_y y) \delta(x)$ excites TE polarized modes with electric field:

$$E_y = \frac{j}{2\beta_x} f(z) \exp(-j\beta_y y) \exp(-j\beta_x x) \quad (1)$$

Since $\delta(x)\delta(y) = \frac{\delta(x)}{2\pi} \int_{-\infty}^{\infty} \exp(-j\beta_y y) d\beta_y$, the field emitted by a point source in the slab may be written, due to linearity, as the superposition of modes excited by line sources with dependence in the form of $\exp(-j\beta_y y)$:

$$E_y = \int_{-\infty}^{\infty} \frac{j}{4\pi\beta_x} f(z) \exp(-j\beta_x x) \exp(-j\beta_y y) d\beta_y \quad (2)$$

In order to calculate this integral, we notice that:

$$H_0^{(2)}(\beta_\rho \rho, \varphi) = \frac{1}{\pi} \int_{\pi/2-j\infty}^{3\pi/2+j\infty} \exp[j\beta_\rho \rho \cos(\zeta - \varphi)] d\zeta \quad (3)$$

where, (ρ, φ, z) is the cylindrical coordination [4]. Using Equation (3), one can show that [appendix 1],

$$\int_{-\infty}^{\infty} \frac{1}{\beta_x} \exp(-j\beta_x x) \exp(-j\beta_y y) d\beta_y = \pi H_0^{(2)}(\beta_\rho \rho) \quad (4)$$

Therefore, a point source given as $E_y = f(z)\delta(x)\delta(y)$, generates a propagating field in the slab in as follows

$$E_y = \int_{-\infty}^{\infty} \frac{j}{4\pi\beta_x} f(z) \exp(-j\beta_x x) \exp(-j\beta_y y) d\beta_y = \frac{j}{4} f(z) H_0^{(2)}(\beta_\rho \rho) \quad (5)$$

As expected, this result coincides with the 2D Green's function $G(x, y) = \frac{j}{4} H_0^{(2)}(k\sqrt{x^2 + y^2})$, solution to

Helmholtz Equation $\nabla^2 G + k^2 G = \delta(x, y)$.

Our goal is to obtain the far field radiation from an array of line sources arbitrarily located along the y axis. In the far-field we can write [5]:

$$H_0^{(2)}(\beta_\rho \rho) \approx \sqrt{\frac{2j}{\pi\beta_\rho \rho}} \exp(-j\beta_\rho \rho) \text{ for } \beta_\rho \rho \gg 0, \quad (6)$$

where $\rho = |\bar{\rho} - \bar{\rho}'| \sim |\bar{\rho}|$ for $|\bar{\rho}| \gg |\bar{\rho}'|$, where, $|\bar{\rho}|$ and $|\bar{\rho}'|$ are the vectors (in the x-y plane) of the observation point and the source, respectively. In the phase term in Equation (6):

$$\rho = |\bar{\rho} - \bar{\rho}'| \sim |\bar{\rho}| - |\bar{\rho}'| \cos(\varphi - \varphi') + \frac{|\bar{\rho}'|^2}{2|\bar{\rho}|} \sim |\bar{\rho}| - |\bar{\rho}'| \cos(\varphi - \varphi') \text{ for } \beta_\rho \frac{|\bar{\rho}'|^2}{2|\bar{\rho}|} \ll 1 \quad (7)$$

Therefore, the conditions to approximate the Green's function with a cylindrical wave in the far field are similar to those applied to spherical waves in 3D (free-space radiation) [6]:

- 1- $\beta_\rho \rho \gg 0$, or the effective wavelength should be much smaller than the observation distance
- 2- $|\bar{\rho}| \gg |\bar{\rho}'|$, or the whole scattering object size (or the array size) should be much smaller than the observation distance
- 3- $\beta_\rho \frac{|\bar{\rho}'|^2}{2|\bar{\rho}|} \ll 1$, for the phase condition.

Of course, there are relevant differences with the far-field radiation in free space: in this ‘flatland’ far-field radiation the local field radiated by a point source appears to be a slab (TE) mode, for which magnetic field and propagation direction are not orthogonal. In other words, the far field is not a TEM wave, for which $\beta^2 = \omega^2 \mu \epsilon$. Without lack of generality, let us consider an observation point in the far field near the x-axis ($y \ll x$, $|z| < h$). We can represent the propagating mode as a transmission-line mode as in [7]. The transmission-line (per unit length) voltages and currents are defined as $V = E_y|_{z=0}$ and $I = -H_z|_{z=0}$, which satisfy the standard transmission line equations:

$$\frac{dV}{dx} = \frac{\partial E_y}{\partial x} \Big|_{z=0} = -j\omega\mu H_z|_{z=0} = j\omega\mu I$$

$$\frac{dI}{dx} = -\frac{\partial H_z}{\partial x} \Big|_{z=0} = j\omega\epsilon E_y|_{z=0} - \frac{\partial H_x}{\partial z} \Big|_{z=0} = j\omega\epsilon_{eff} V$$

Where

$$\epsilon_{eff} = \epsilon - \frac{\partial H_x / \partial z|_{z=0}}{j\omega\epsilon E_y|_{z=0}} \quad (8)$$

Thus, the effective permittivity of the mode is modified by presence of a longitudinal component of magnetic field H_x . The dispersion relation for the modes in the far field is compactly written in analogy with the regular free-space radiation:

$$\beta_\rho^2 = \omega^2 \mu \epsilon_{eff} . \quad (9)$$

In other words, within this framework, it is possible to treat the radiation from arbitrary point sources within the slab (2D) as the one of point sources in free-space (3D), by simply considering an effective form of permittivity as defined in Equation (8), similar to the concept of ‘‘flatland optics’’ [3].

Note that, the definition of point source in the form of an impressed field given above lets us directly derive the field radiated from a waveguide interface into the slab region. In Fig. 1(b), at $x=0$, where the array waveguides are connected to the slab waveguide, the waveguides’ facets may be treated as collections of 2D point sources defined above, and thus, following the previous theoretical formulation, the overall radiated field inside the slab may be described as the one produced by an optical phased array. In particular, on the plane $x=0$ the field may be calculated as:

$$\sum_{n=1}^N \int_{-W_e/2}^{W_e/2} \frac{jA_n}{4\pi} \exp(-j\varphi_n) H_0^{(2)} \left(\omega \sqrt{\mu \epsilon_{eff}} \left| \bar{\rho} - \bar{d}_n - y \right| \right) dy \quad (10)$$

where, N is the number of array elements, A_n is the field amplitude of the n^{th} array waveguide, W_e is the effective width of a waveguide, including the penetration depth due to the Goose-Hahnchen shift, \bar{d}_n is the position (center) of the array elements, φ_n is the input phase of the n^{th} array waveguide. Also, ϵ_{eff} is the effective permittivity defined in Equation (8). This formula can be simplified as shown in Equation (7) in the far field zone.

3. CONCLUSIONS

We have presented the far-field modeling of two-dimensional (2D) propagation in slab waveguides. The far field conditions and field formulations have been derived. Our results can be used to model field propagation inside a thin slab waveguide, which is excited by an array of waveguides, such as the outputs of multimode interference (MMI) beam splitters [8,9].

Appendix 1

We derive here an integral representation of the 2-D Hankel function of use to derive Equation (4) above. We know that [4]:

$$H_0^{(2)}(\beta_\rho \rho, \varphi) = \frac{1}{\pi} \int_{\pi/2 - j\infty}^{3\pi/2 + j\infty} \exp[j\beta_\rho \rho \cos(\zeta - \varphi)] d\zeta.$$

One can write

$$\begin{aligned} H_0^{(2)}(\beta_\rho \rho, \varphi) &= \frac{1}{\pi} \int_{0 - j\infty}^{\pi + j\infty} \exp[-j\beta_\rho \rho \sin(\zeta - \varphi)] d\zeta = \\ &= \frac{1}{\pi} \int_{0 - j\infty}^{\pi + j\infty} \exp[-j\beta_\rho \rho \sin(\zeta) \cos(\varphi)] \exp[j\beta_\rho \rho \cos(\zeta) \sin(\varphi)] d\zeta = \\ &= \frac{1}{\pi} \int_{0 - j\infty}^{\pi + j\infty} \exp[-j\beta_\rho x \sin(\zeta)] \exp[j\beta_\rho y \cos(\zeta)] d\zeta. \end{aligned}$$

By changing the variables as $\beta_x = \beta_\rho \sin(\xi)$ and $\beta_y = -\beta_\rho \cos(\xi)$, one can show

$$H_0^{(2)}(\beta_\rho \rho, \varphi) = \frac{1}{\pi} \int_{\xi=0 - j\infty}^{\xi=\pi + j\infty} \exp[-j\beta_\rho x \sin(\xi)] \exp[j\beta_\rho y \cos(\xi)] \frac{1}{\sqrt{\beta_\rho^2 - \beta_x^2}} d\beta_x$$

Since

$$\begin{aligned} \pi < \xi < \pi + j\infty &\rightarrow -j\infty < \beta_x < 0j \text{ and } \beta_\rho < \beta_y < \infty \\ \pi/2 < \xi < \pi &\rightarrow 0 < \beta_x < \beta_\rho \text{ and } 0 < \beta_y < \beta_\rho \\ 0 < \xi < \pi/2 &\rightarrow 0 < \beta_x < \beta_\rho \text{ and } -\beta_\rho < \beta_y < 0 \\ -j\infty < \xi < 0 &\rightarrow -j\infty < \beta_x < 0j \text{ and } -\infty < \beta_y < -\beta_\rho \end{aligned}$$

Thus

$$H_0^{(2)}(\beta_\rho \rho, \varphi) = \frac{1}{\pi} \int_{\beta_y=-\infty}^{\beta_y=\infty} \exp[-j\beta_x x] \exp[j\beta_y y] \frac{1}{\sqrt{\beta_\rho^2 - \beta_x^2}} d\beta_x$$

Therefore, we can conclude:

$$\int_{-\infty}^{\infty} \frac{1}{\beta_y} \exp(-j\beta_x x) \exp(-j\beta_y y) d\beta_x = \pi H_0^{(2)}(\beta_\rho \rho)$$

REFERENCES

- [1] M. Jarrahi, R.F.W. Pease, D.A.B. Miller, T. H. Lee, T.H., "Optical Spatial Quantization for Higher Performance Analog-to-Digital Conversion," *IEEE Trans. Microwave Theory and Techniques* 56(9), 2143-2150 (2008).
- [2] R. Zia, M. L. Brongersma, "Surface plasmon polariton analogue to Young's double-slit experiment," *Nat Nanotechnol.* 2(7), 426-429. (2007).
- [3] Adolf W. Lohmann, Avi Pe'er, Dayong Wang, and Asher A. Friesem, "Flatland optics: fundamentals," *J. Opt. Soc. Am. A* 17, 1755-1762 (2000).
- [4] A. Sommerfeld, "Partial Differential Equations in Physics," New York: Academic (1949).
- [5] M. Abramowitz, I. A. Stegun, Irene, "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables," New York: Dover Publications (1972).
- [6] C. A. Balanis, "Advanced Engineering Electromagnetics," New York: Wiley (1989).
- [7] A. Alù and N. Engheta, "Optical nanotransmission lines: synthesis of planar left-handed metamaterials in the infrared and visible regimes," *J. Opt. Soc. Am. B* 23, 571-583 (2006).
- [8] A. Hosseini, H. Subbaraman, D. Kwong, Y. Zhang, and R. T. Chen, "Optimum access waveguide width for $1 \times N$ multimode interference couplers on silicon nanomembrane," *Opt. Lett.* 35, 2864-2866 (2010).
- [9] A. Hosseini, D. N. Kwong, C.-Y. Lin, B. S. Lee, and R. T. Chen, "Output Formulation for Symmetrically-Excited one-to- N Multimode Interference Coupler," *IEEE J. of Sel. Topics in Quant. Elect.* 6(1), 53-60 (2010).