

# Branch and bound methods based tone injection schemes for PAPR reduction of DCO-OFDM visible light communications

YONGQIANG HEI,<sup>1,\*</sup> JIAO LIU,<sup>1</sup> WENTAO LI,<sup>2</sup> XIAOCHUAN XU,<sup>3</sup> AND RAY T. CHEN<sup>3</sup>

<sup>1</sup>State Key Laboratory of Integrated Service Network, Department of Communication Engineering, Xidian University, Xi'an 710071, China

<sup>2</sup>Science and Technology on Antenna and Microwave Laboratory, Department of Electronic Engineering, Xidian University, Xi'an 710071, China

<sup>3</sup>Microelectronics Research Center, Department of Electrical and Computer Engineering, University of Texas at Austin, Austin, TX 78758 USA

\*yqhei@utexas.edu

**Abstract:** In this paper, the peak-to-average power ratio (PAPR) reduction problem with tone injection (TI) in the direct current-biased optical orthogonal frequency division multiplexing (DCO-OFDM) system is investigated, which is formulated as a tough integer combinatorial optimization problem. Since it is a challenging task to find the global optimal solution, the branch-and-bound method (BBM), which is extensively employed to deal with the NP-hard problem, is introduced to solve this problem. By splitting the superior branches and pruning the inferior branches, the close optimal solution is obtained. Simulation results reveal that the proposed BBM-based TI method has superior PAPR reduction and bit error rate (BER) performance compared with some existing algorithms. The proposed algorithm ensures a relatively low peak value, and thus provides an important benchmark in performance evaluations relative to other existing algorithms targeting at the same problem.

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## 1. Introduction

Owing to the advantages of huge bandwidth, unlicensed frequency, immune to electromagnetic interference and high security, visible light communications (VLC) system has attracted considerable attention recently. Unlike radio frequency (RF) communications, VLC with white light emitting diodes (LEDs) can provide both illumination and communication simultaneously, and has the merit of high efficiency, long life, low cost and power consumption. Hence, it is considered as one promising enabling technology for future wireless communications. VLC can be implemented by using on-off Keying (OOK), variable pulse position modulation (VPPM) or pulse amplitude modulation (PAM) [1]. However, these modulation techniques can be easily affected by the inter symbol interference (ISI). Nowadays orthogonal frequency division multiplexing (OFDM) is more favored in VLC [2–4] system because it cannot only combat the ISI induced by multi-path, but also enhance the spectrum efficiency. One possible method to introduce OFDM into the VLC system is the DC-biased optical OFDM (DCO-OFDM) system, which guarantees the positive real inputs by adding the DC bias. However, one intrinsic issue in the DCO-OFDM system is its high peak-to-average power ratio (PAPR). The signals with high PAPR passing through power amplifier and light emitting diode (LED) [5] emitter can result in a nonlinear distortion. Furthermore, the high signal peak value requires a large DC bias in DCO-OFDM system, which causes serious degradation of system power efficiency. Thus, PAPR reduction in VLC OFDM systems is very crucial.

Due to the difference in the transmitted signals, traditional PAPR reduction methods in RF-OFDM systems cannot be directly applied in VLC OFDM systems. In [6], an iterative clipping method ensuring no in-band distortion is proposed to reduce PAPR in VLC OFDM systems. An exponential nonlinear companding technique is proposed in [7] for VLC OFDM systems, which reduces the PAPR by compressing large signals and expanding the small signals simultaneously. Nevertheless, both methods mentioned above can result in the BER performance degradation caused by the nonlinear distortion of signals. Additionally, a pilot-assisted technique for PAPR reduction based on the selected mapping (SLM) algorithm is presented for an optical OFDM system in [8], which is demonstrated to be better than conventional SLM methods in terms of PAPR reduction for high order constellations. Unfortunately, the pilot-assisted method probably results in data rate loss because of the expense of the pilot. In RF-OFDM systems, the tone injection (TI) technique is regarded as a distortionless technique which can reduce PAPR significantly without transmission of any side information and data rate loss. An optimized TI strategy with semidefinite relaxation (SR-TI) is proposed in [9] to reduce PAPR for DCO-OFDM system, in which the final solution for the relaxed PAPR reduction optimization problem is generated by randomization method based on the optimized positive semi-definite matrix. However, the optimization model after relaxation becomes a high complexity second-order cone programming (SOCP). In order to further reduce its complexity, the linear programming based TI algorithm (LP-TI) [10] is proposed for PAPR reduction.

In this paper, the PAPR reduction problem based on TI in DCO-OFDM systems is formulated as a combinatorial optimization problem. Then, a branch-and-bound method (BBM), which is widely used to solve the NP-hard problem in wireless communications [11]

and achieve the close global optimal solution in power systems [12], is employed to find the close optimal solution. The proposed scheme starts with getting a feasible solution, and then splits some branches with higher probability to obtain better solution and prunes the other branches with lower probability to obtain better solutions. The iterative algorithm keeps splitting and pruning until the termination condition is satisfied. Finally, the close best solution is achieved and it is taken as the final optimal solution. Simulation results validate that the proposed BBM scheme can achieve a superior PAPR reduction performance compared with the existing TI schemes.

## 2. System model and problem formulation

Consider a DCO-OFDM VLC system as shown in Fig. 1. Since the VLC system is based on intensity modulation and direct detection techniques, the OFDM signal to modulate the LED must be real and unipolar. These requirements can be satisfied by the Hermitian symmetry property in the frequency domain and adding direct current (DC) bias to the time domain signal. At the transmitter, by using the inverse Fast Fourier Transform (IFFT), a set of multiplexed subcarriers in the frequency domain is transformed to its time-domain equivalent signal. Then, a cyclic prefix (CP) is added and the resulting signal is digital to analog converted (DAC) to obtain a bipolar time-domain real-valued OFDM signal. After adding a DC bias to the time domain signal, the unipolar signal drives the LED to convert the electrical signals to the optical signals. At receiver, the optical signals are first converted to the electrical signals by photodiode. Then a reverse process can be implemented to demodulate the data.

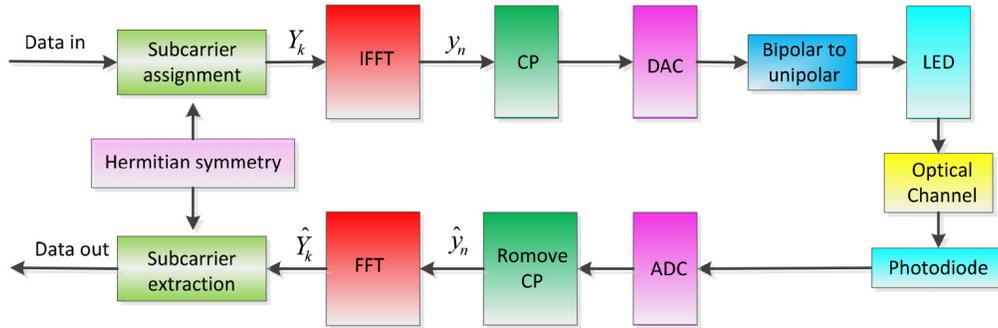


Fig. 1. The schematic diagram of the DCO-OFDM VLC system.

Denote  $\{X_k\}_{k=0}^{N/2-1}$  as the original input data, where  $N$  is the number of the subcarriers. Then the discrete-frequency signals  $\mathbf{X}=[X_0, X_1, \dots, X_{N-1}]$  should satisfy the following constraint:

$$X_k = \begin{cases} X_k & k = 1, 2, \dots, N/2-1 \\ X_{N-k}^* & k = N/2+1, \dots, N-1 \end{cases} \quad (1)$$

where  $X_0 = X_{N/2} = 0$  and  $(\cdot)^*$  represents the conjugation operation. Then, the equivalent time-domain OFDM baseband signal over-sampled  $L$ -times is expressed as:

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/NL}, n = 0, 1, 2, \dots, LN-1. \quad (2)$$

The PAPR of the OFDM signal is given by:

$$PAPR = 10 \log_{10} \frac{\max_{0 \leq n \leq N-1} |x_n|^2}{E[|x_n|^2]}, \quad (3)$$

where  $E[\cdot]$  denotes the expected value operation.

The basic concept of the tone injection (TI) in reducing the PAPR is to expand the original  $M$ -ary quadrature amplitude modulation (QAM) constellation by mapping each point into several different constellation points [13]. For convenience, the expanded constellation in [14] is adopted in this paper and it maps one constellation point to the outer ring of the original constellation. For better understanding, one example of the extended 16-QAM constellation is shown in Fig. 2, in which the constellations inside the dashed box are the original ones, while the constellations outside the dashed box are the extension ones of the original constellations. Due to the extended constellation points do not overlap with the original positions, this TI scheme does not require any side information. Hence, the received symbol  $\hat{Y}_k$  after fast Fourier Transform (FFT) can be directly decoded in accordance with the extended constellation.

After applying TI in the VLC OFDM system, the signal can be further written as:

$$\bar{X}_k = (1 - \omega_k) X_k + \omega_k C_k, \quad (4)$$

$$C_k = \begin{cases} -(d/2)a_k - j(d/2)M'', & (a_k > -M', b_k = M') \\ -(d/2)a_k + j(d/2)M'', & (a_k < M', b_k = -M') \\ -(d/2)M'' - j(d/2)b_k, & (a_k = M', -M' < b_k < M') \\ (d/2)M'' - j(d/2)b_k, & (a_k = -M', -M' < b_k < M') \\ -(d/2)a_k - j(d/2)M'', & (a_k = -M', b_k = M') \\ -(d/2)a_k + j(d/2)M'', & (a_k = M', b_k = -M') \end{cases} \quad (5)$$

where  $d$  is the minimum distance between two adjacent points,  $a_k$  and  $b_k$  are integer multiples of  $d/2$  taking values from the set  $\{\pm 1, \pm 3, \dots, \pm \sqrt{M}-1\}$ , i.e.,  $X_k = (d/2)a_k + j(d/2)b_k$ ,  $M' = \sqrt{M}-1$  and  $M'' = \sqrt{M}+1$ . With the conjugate symmetry operation, the symbols assigned to OFDM subcarriers are expressed as:

$$Y_k = \begin{cases} \Re\{\bar{X}_0\}, & k=0 \\ \bar{X}_k, & k=1, \dots, \frac{N}{2}-1 \\ \Im\{\bar{X}_0\}, & k=\frac{N}{2} \\ \bar{X}_{N-k}^*, & k=\frac{N}{2}+1, \dots, N-1 \end{cases}, \quad (6)$$

where  $\Re\{\cdot\}$  and  $\Im\{\cdot\}$  represent the real part and the imaginary part respectively.

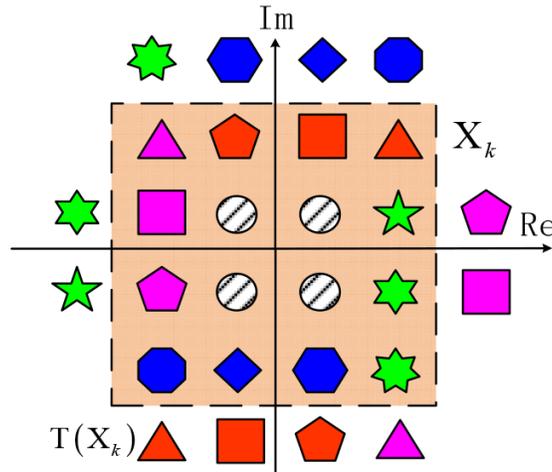


Fig. 2. The extended 16-QAM constellation diagram.

Furthermore, by applying inverse fast Fourier transform (IFFT) to  $\{Y_k\}$ , the discrete-time signals over-sampled  $L$ -times can be written as:

$$y_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} Y_k e^{j2\pi kn/LN}, n = 0, 1, 2, \dots, LN - 1. \quad (7)$$

Since there only exists one equivalent constellation for each original point, the PAPR optimization problem can be expressed as:

$$\begin{aligned} \min_{\omega} \quad & C_{sel}(\omega) \\ C_{sel}(\omega) = \quad & \max |y_n(\omega)|^2 \\ \text{subject to} \quad & \omega \in \{0, 1\}^N, \end{aligned} \quad (8)$$

where  $\{0, 1\}^N$  stands for a  $N$ -dimensional binary sequence. The problem in Eq. (8) is a tough integer optimization task with the complexity of  $O(2^N)$ .

### 3. The BBM-based TI scheme for PAPR reduction

In this section, we focus on how to address the PAPR reduction problem in Eq. (8) by BBM. The specific details are provided below.

#### 3.1 Branch

To clarify the BBM method, we first define some symbols first. Let  $Q_j$  denote the  $j$ -th region generated by rounding off non-integral elements in  $\omega$  to 0 or 1.  $LB(Q_j)$  represents the lower bound of the optimization problem.

Initially, i.e.  $t = 0$ ,  $Q_0 = \{\omega \mid \omega_k \in [0, 1], \forall k\}$ . By solving

$$\begin{aligned} \min_{\omega} \quad & C_{sel}(\omega) \\ \text{s.t.} \quad & Q_0. \end{aligned} \quad (9)$$

The lower bound  $LB(Q_0)$  can be obtained with interior-method. Then,  $Q_0$  is split into two regions  $Q_1$  and  $Q_2$  according to a non-integral element in  $\omega$  (say,  $\omega_l$ ) at step  $t=1$ . Correspondingly, two new sub-problems generated by  $Q_1$  and  $Q_2$  are expressed as follows:

$$\begin{aligned} \min_{\omega} C_{sel}(\omega) \\ \text{s.t. } Q_1 : \omega_k \in [0,1], \forall k \neq l, \omega_l = 0, \end{aligned} \quad (10)$$

$$\begin{aligned} \min_{\omega} C_{sel}(\omega) \\ \text{s.t. } Q_2 : \omega_k \in [0,1], \forall k \neq l, \omega_l = 1. \end{aligned} \quad (11)$$

By addressing Eq. (10) and Eq. (11) with interior-point method, two new lower bounds  $LB(Q_1)$  and  $LB(Q_2)$  are obtained respectively. Then, we select one of  $Q_1$  and  $Q_2$  as lower bound to split along an undetermined element  $\omega_k$  ( $k \neq l$ , the element  $\omega_l$  is a fixed integer and the index  $l$  is employed) for the next step. There are only  $k-2$  non-integer variables after addressing the two new sub-problems. This splitting process will not stop until the termination criterion is satisfied.

In BBM, there are two key issues required to be defined. One is the branch with the smaller lower bound to split at each step, from which the better solution with the highest probability can be found. The other is the rule to select which element to be split at the  $k$ -th step. In this paper, it is defined as follows:

$$k^* = \arg \min_{k \in \{1, \dots, N\}} \left| \omega_k - \frac{1}{2} \right|. \quad (12)$$

The rule of Eq. (12) can be explained as follows, for each branch, the region is split along the value which is the closest to  $1/2$  (most uncertain point in  $[0,1]$ ).

### 3.2 Bound

In each iteration, BBM requires to compute the lower bound  $LB(Q_j)$  for each branch. From the previous subsection, we know that  $LB(Q_j)$  can be obtained by directly solving the corresponding optimization problem with relaxed  $Q_j$ . For instance,  $LB(Q_0)$  is obtained by relaxing  $Q_0$  to  $\{0 \leq \omega_k \leq 1, \forall k\}$  in Eq. (9) and  $LB(Q_1)$  is obtained by relaxing  $Q_1$  to  $\{0 \leq \omega_k \leq 1, \forall k \neq l, \omega_l = 0\}$  in Eq. (10). Besides, the feasibility of solutions to Eq. (8) can be determined by the criterion that each element of  $\omega$  is an integer. After obtaining a feasible solution, we will prune the branch according to the following rules.

- Set the initial upper bound  $U = +\infty$ . Suppose a feasible solution  $Q_h$  which is obtained after the  $T$ -th iteration;
- If each element of  $\omega$  is an integer and  $LB(Q_h) < U$ ,  $Q_h$  becomes a candidate solution of Eq. (8) and set  $U = LB(Q_h)$ ;
- If  $\omega$  contains at least one non-integral element and  $LB(Q_h) < U$ , it implies that the smaller solution may be contained in this branch. Then find the feasible solution  $LB(Q_m)$  of this branch;
- If  $LB(Q_h) > U$ , prune this branch. This is because there is low probability to obtain any other better solution included in this branch than the current one.

Jointly considering branch and bound structure in BBM, the detailed procedures to optimize the PAPR problem in DCO-OFDM systems are provided by Algorithm 1 below.

**Algorithm 1: BBM-based TI method for PAPR reduction.**

**Step 1 Initialization:**

- Set the initial upper bound  $U = +\infty$  and iteration index  $T = 1$  ;
- Set the initial region of branches required to be split as  $Q_0$ . If all elements of the solution in  $\Theta$  are integers, stop and take  $\Theta$  as the final solution. Otherwise, split along the undetermined elements until a feasible solution is obtained according to the branching operation in subsection 3.1;
- Set  $U = LB(Q_h)$ , where  $Q_h$  represents the region of feasible solution.

**Step 2** When the node without branch exists,

- Prune the branch  $Q$  which satisfies  $LB(Q) > U$  ;
- Select the region with index  $h^*$  which has the non-integral elements and the best lower bound to split;
- Determine the split index  $k^*$  based on Eq. (12) for the selected region  $Q_{h^*}$  ;
- Split  $Q_{h^*}$  into two regions  $Q_{2T}$  and  $Q_{2T+1}$  based on  $\omega_{k^*}$ . Continue to split one of  $Q_{2T}$  and  $Q_{2T+1}$  with lower bound until a feasible solution  $\Theta$  is obtained. The region of feasible solution  $\Theta$  is denoted as  $Q_h$  ;
- If  $LB(Q_h) > U$ , prune the branch corresponding to  $Q_{h^*}$ . Otherwise, set  $U = LB(Q_h)$  and prune the original branch;
- $T = T + 1$ .

**Step 3** Take  $\Theta$  as the final optimal solution corresponding to the peak value  $U$ .

#### 4. Simulation results

To evaluate the proposed BBM-based TI scheme in terms of PAPR reduction in VLC OFDM systems, the complementary cumulative distribution function (CCDF) is used. In the simulation, we assume that DCO-OFDM systems have  $N = 128$  subcarriers, in which the transmit signals are modulated by 16-QAM and oversampled factor is set as  $L = 4$ . Classical methods, such as the semidefinite relaxation (SR)-TI method in [9] and the linear programming (LP)-TI method in [10] for PAPR reduction are also simulated in the VLC OFDM system for comparison. Both the SR-TI method [9] and the LP-TI method [10] method use the extended 16-QAM constellation with higher degrees of freedom than that of the BBM-based TI method in order to testify its effectiveness and efficiency. Additionally, the iteration number in SR-TI and LP-TI is selected as 128 since no obvious performance improvement can be observed after 128 iterations.

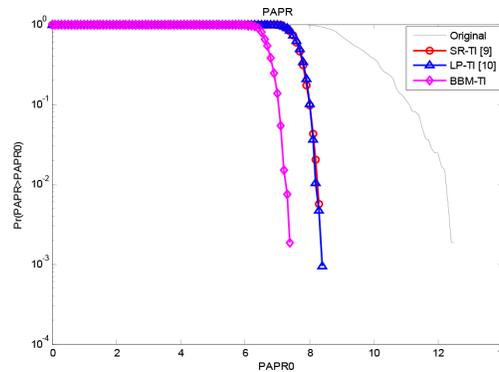


Fig. 3. PAPR reduction comparison of different TI methods with  $N = 128$  and 16-QAM.

In Fig. 3, simulation results in terms of the PAPR CCDF for different TI methods are provided. At  $10^{-2}$ , observing that BBM-based TI obtains about 5 dB PAPR reduction over the original OFDM system. Besides, it performs better than the SR-TI and the LP-TI methods in terms of PAPR reduction. Compared with 4 equivalent constellation points in SR-TI and 9 equivalent constellation points in LP-TI for PAPR reduction, BBM-based TI has less than one equivalent constellation point on average. Hence, the good PAPR reduction performance indicates the promising ability of the proposed BBM-based TI method in dealing with the PAPR optimization problem.

To further evaluate the BBM-based TI method on the PAPR reduction in VLC OFDM systems, over 5000 OFDM signals are randomly generated to compare the peak power and the mean power achieved by BBM-based TI, LP-TI and SR-TI. The corresponding results are plotted in Fig. 4 and Fig. 5, respectively. For reference, the peak and the mean power of the original OFDM signals are also provided. It is noticed that the peak power by BBM-based TI method is much lower. From Fig. 5, we know that the BBM-based TI method consumes smaller energy compared with the LP-TI method and the SR-TI method, which therefore proves the effectiveness of the proposed BBM-based TI method.

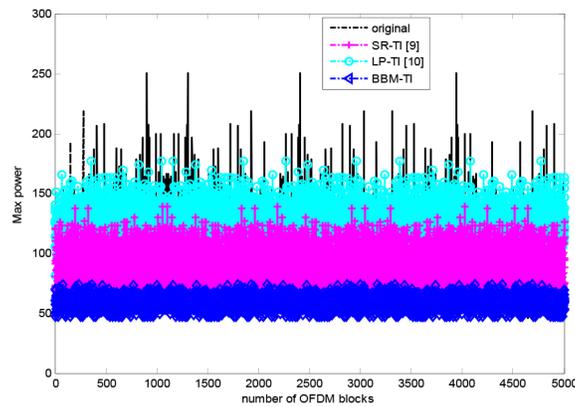


Fig. 4. Peak power comparison of different TI methods.

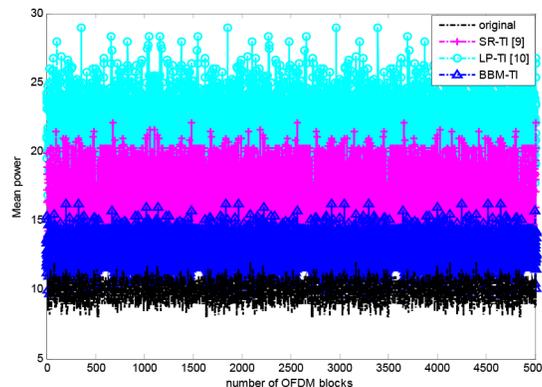


Fig. 5. Mean power comparison of different TI methods.

In this paper, a simplified LED nonlinear model is employed in the DCO-OFDM system with the linear LED range of  $[0, A_m]$ . To assure a steady light intensity and maximize the modulation depth, the DC bias is set to  $A_m/2$ . Then the DCO-OFDM signals are expressed as:

$$y_{n,DCO} = \begin{cases} A_m, & y_n > A_m \\ y_n, & 0 \leq y_n \leq A_m \\ 0, & y_n \leq 0 \end{cases} \quad (13)$$

Essentially, LED will degrade the BER performance of OFDM system due to the non-linear distortion. Hence, PAPR reduction technique is employed to mitigate the effect of non-linear distortion caused by LED. Figure 6 shows BER performance comparison among different TI schemes under the additive white Gaussian noise (AWGN) channel with LED. The simulation results indicate that the BBM-based TI scheme not only obtains significant PAPR reduction, but also effectively reduces the nonlinear distortion and hence has better BER performance over LP-TI and SR-TI in AWGN channel.

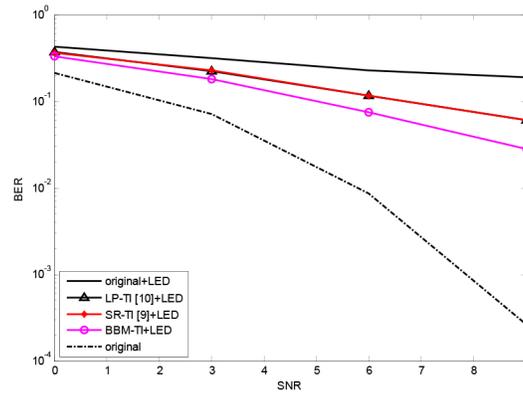


Fig. 6. BER comparison of different TI methods with  $N = 128$  and 16-QAM.

**Complexity evaluation:** when taking the practical implementation into account, one important factor that has to be considered is the complexity of the TI scheme, which directly determines whether this scheme can be applied or not. According to [10], the complexity of linear programming problem is  $O(N^3 RL)$ , where  $N$  is the number of subcarriers,  $R$  denotes the total number of rotations and  $L$  is over-sampling size. In [9], the PAPR problem is formulated as a semi-definite programming (SDP) problem and then cast as a second-order cone programming (SOCP) problem, which is because SOCP runs far more efficiently than SDP and has a much better worst-case complexity. According to [15], the SOCP problem solved by the interior-point method has a complexity of  $O((N+1)^7)$  per iteration. However, if using the primal-dual method to deal with the SOCP problem, the accuracy of a given solution can be improved by an absolute constant factor in  $O((N+1)^{3/2})$  iterations [16]. In that case, the total complexity of the SOCP problem is  $O((N+1)^{8.5})$ . Hence, the semidefinite relaxation (SR) algorithm [9] is much more complex than the linear programming (LP) method [10]. For the proposed BBM method, it can be efficiently solved by the interior-point method, which has a complexity of  $O(C \times \max((N/2-1)^3, (N/2-1)^2(N-2), F))$  in solving the PAPR optimization problem (8), where  $C$  denotes the number of steps,  $N/2-1$  represents the dimension of the variable  $\omega$ ,  $N-2$  is the number of constraints and  $F$  is the cost in evaluating the first and the second derivatives of the objective and constraint functions [17]. Since (8) has only one objective function that can be translated into real variable for each element of  $\omega$ ,  $F$  can be ignored. Hence, the whole complexity of the BBM method by using the interior-point method is  $O(CN^3/4)$  per iteration. Therefore, the BBM method owns a lower complexity than the SR algorithm [9] per iteration. Since the interior-point method in BBM-TI is responsible for

finding the local optimal solution of the PAPR optimization problem (8), the number of steps  $C$  can be much smaller when compared with the case of finding a global optimization solution. More importantly, the interior-point method in the BBM-TI method can be replaced by other lower complexity methods, provided that they can find the same local optimal solution. That is to say, the complexity of the proposed BBM-based TI method can be further reduced. In conclusion, the proposed BBM-TI method has better PAPR reduction performance than the SR-TI algorithm [9] and the LP-TI algorithm [10], and lower computational complexity when compared with the SR-TI algorithm [9].

## 5. Conclusion

In this paper, an efficient BBM-based TI method proposed for PAPR reduction in VLC DCO-OFDM systems is presented. The PAPR reduction problem is formulated as a tough integer combinatorial optimization problem, and a widely used method to solve NP-hard problem, i.e., branch-and-bound method (BBM) is introduced to deal with this problem. Simulation results showed that the proposed scheme can achieve significant PAPR reduction performance and better BER performance for VLC system compared with the existing TI schemes.