# A linear electro-optic modulator based on an array of Mach-Zehnder interferometers

Zhou Z. Yue[a], Suning Tang[b], Ray T. Chen[a]

[a]Microelectronic Research Center, Department of Electrical and Computer Engineering,
The University of Texas at Austin, Austin, Texas 78758-4445.
[b]Radiant Research, Inc., 9430 Research Blvd., Suite IV-305, Austin, TX 78759-6543

## ABSTRACT

A linearized external electro-optic modulator with a number of parallel Mach-Zehnder interferometers cascaded via directional couplers is proposed. Fast-Fourier-Transform method is used to analyze both the third harmonic spurious signal and the third intermodulation distortion. By adjusting the number of Mach-Zehnder interferometers contained in the modulator, the linearity of the transfer curve of the modulator can be modified while obtaining nearly 100% optical modulating depth. The modulator is based on an innovative domain-inverted electro-optic polymer film, where the two arms of each Mach-Zehnder interferometer are inversely poled. As a result, only one driving voltage is required upon a pair of uniform traveling-wave electrodes, for high-speed operation.

**Key words:** electro-optic modulator, Mech-Zehnder interferometer, linearity

## 1. INTRODUCTION

Analog transmissions over fiber-optic medium, such as cable-TV distributions and microwave applications, requires a linear electro-optic device, i.e., either analog lasers or linearized external modulators.[1-6] The operating speed of electrically modulated lasers is limited because of the instability of the lasers. External modulators, either the discrete interference type Mach-Zehnder modulators or the distributed interference type directional-coupler modulators, have many promising advantages such as no second-order signal distortion, but they are also limited by their inherent nonlinearity due to the intrinsic sine-squared type of driving voltage dependence. Various modulator configurations have been proposed and investigated in the last few years to improve the linearity of external modulators.[7-14] The two most common configurations are parallel arranged interferometers and sequentially cascaded interferometers. Both of them are attempted to synthesize a linear transfer curve and to eliminate the cubic term in the Taylor-Series-Expansion of the transfer function. The parallel configuration, like the dual-parallel Mach-Zehnder and its variations, has shown successful improvement in modulator's linearity, but they usually require a precise control of the multiple input optical signals and multiple driving sources.[15-17] The various sequential cascading of Mach-Zehnder interferometers and/or directional couplers feed forward the one input optical signal, and thus have not such kind of problems, and have also shown significantly linearized transfer curve, but the spur-free optical modulation index is limited. As a result, a large portion of the input optical power must be sacrificed as a trade off for achieving good modulation linearity.[18-24]

We propose a new configuration of linearized modulator which is composed of a number of Mach-Zehnder interferometers parallel cascaded via directional couplers. The Mach-Zehnder interferometers with different length emulate the first several terms of the Fourier transform of a triangular function, so that the linearity of the modulator can be obtained by adjusting the number of terms used in the Fourier series or the number of Mach-Zehnder interferometers integrated in the modulator. Directional couplers with different length passively distribute the input light power between the Mach-Zehnders, so that only one optical input signal is needed. An optical DC bias is offered by the first directional coupler, the spurious-free optical modulation range can be as high as ~100%. Domain inverted electro-optic polymers are employed to fabricate the modulator, so that only one driving voltage and a pair of uniform traveling-wave electrodes is needed, which guarantees the high-speed operation of the modulator. Fourier-spectrum-analysis method is employed to analyze both the third-harmonic spurious signal and the 3rd-order-intermodulation distortion. The Fourier transform of the modulated optical signal includes all the high orders of spurious signal. It has shown much more precise results than those obtained by the conventional Taylor-series-approximation method, especially when the linearity of the modulator is close to ideal, and the contribution of the higher orders spurious signal can not be neglected.[25-26]

## 2. DEVICE DESIGN

Fig. 1 shows an ideal transfer curve of a linear electro-optic modulator, which is actually a triangular function

$$H_{(V)} = V / V_{max} \qquad (0 \le V \le V_{max}) \tag{1}$$

where V is the modulating voltage and $V_{max}$ is the peak value of the modulating voltage.

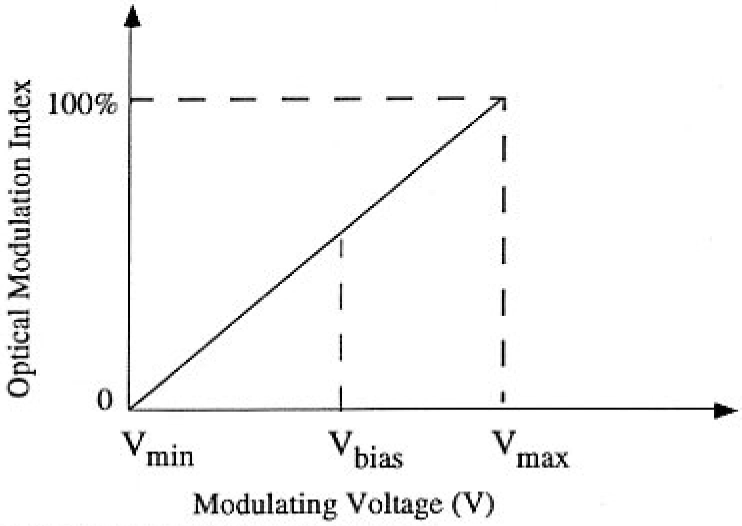

Fig. 1. An ideal transfer curve of an electro-optic modulator.

The Fourier transform of this triangular transfer function can be expressed as:

$$H_{(V)} = \frac{1}{2} - \frac{4}{\pi^2}\left[\cos\left(\frac{\pi V}{V_{max}}\right) + \frac{1}{3^2}\cos\left(\frac{3\pi V}{V_{max}}\right) + \frac{1}{5^2}\cos\left(\frac{5\pi V}{V_{max}}\right) + \ldots\right]. \tag{2}$$

To achieve an ideal transfer curve as shown in Fig.1, the only thing is to simulate each term in Eq.(2) using optical devices. But since we can only realize a limited number of terms in Eq.(2), in order to obtain the maximum optical modulation depth, the index in front of the parentheses in Eq. (2) can be replaced with

$$a = \frac{1}{2}\left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\right)^{-1}, \tag{3}$$

so that Eq. (2) changed to

$$H_{(V)} = \frac{1}{2} - a\left[\cos\left(\frac{\pi V}{V_{max}}\right) + \frac{1}{3^2}\cos\left(\frac{3\pi V}{V_{max}}\right) + \frac{1}{5^2}\cos\left(\frac{5\pi V}{V_{max}}\right) + \ldots\right]. \tag{4}$$

Eq. (4) represent an approximation of the ideal transfer curve in Fig. (1). In this equation, the more terms remain not being neglected, the better the linearity of the represented transfer curve.

The proposed linearized modulator is shown in Fig. (2), which exactly embedded the transfer curve of Eq. (4). The passive directional couplers with different length distribute the input light power between the Mach-Zehnder interferometers and the DC optical bias. Each Mach-Zehnder interferometer realizes a cose(θ) item in Eq. (4).

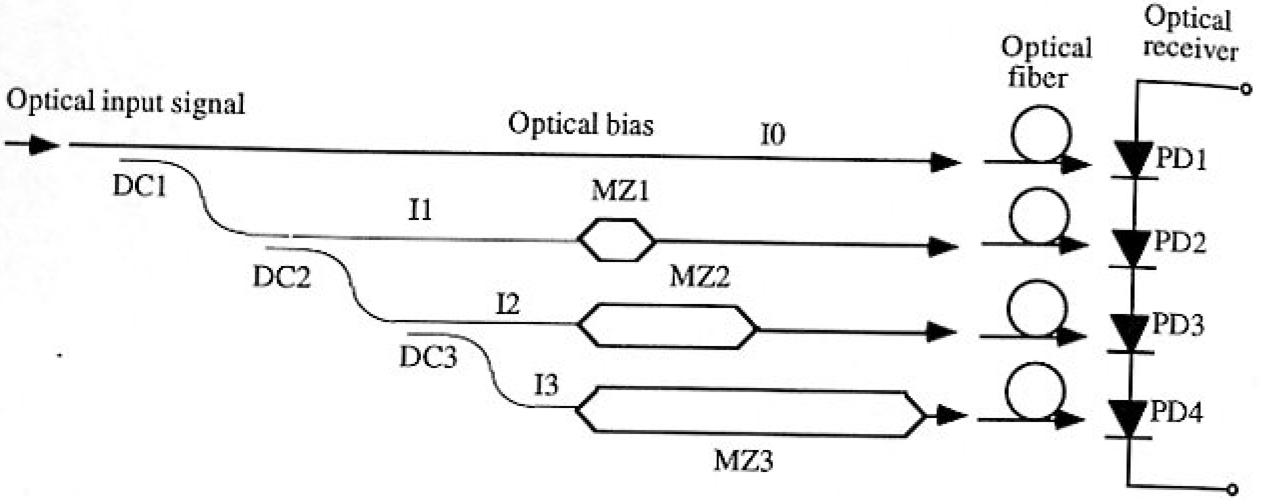

Fig. 2. Linearized electro-optic modulator. Mach-Zehnder interferometers MZ1, MZ2, MZ3... are parallel cascaded via directional couplers DC1, DC2, DC3....

One of the Mach-Zehnder interferometers used in this design is shown in Fig. 3. The modulator is fabricated on an electro-optic polymer film, the two arms of the Mach-Zehnder interferometer are poled in opposite directions. After poling, the two arms of the interferometer are covered by two pieces of uniform traveling-wave electrodes on the top and bottom. with phase retardation between the two arms $\Delta\Phi$ and input light intensity $I_{in}$, the output light intensity is

$$I_{out} = \frac{I_{in}}{2}[1 - \cos(\Delta\Phi)]. \tag{5}$$

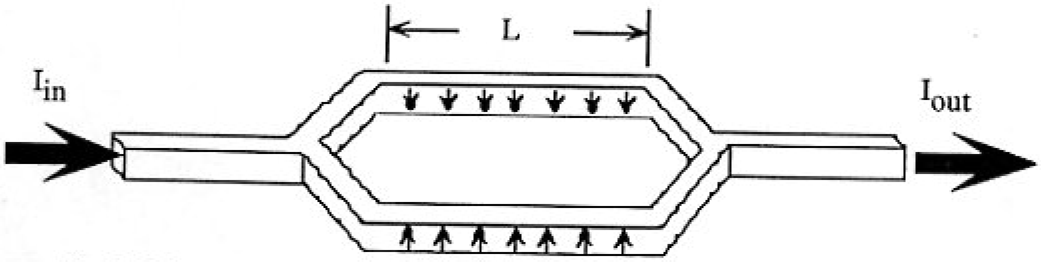

Fig. 3. Schematic diagram of the Mach-Zehnder interferometer used in this design.

With polymeric electro-optic coefficient $\gamma_{33}$ and applied driving voltage V, the phase retardation of the two arms is

$$\Delta\Phi = 2\pi n^3 \gamma_{33} VL / \lambda d, \tag{6}$$

where n is the refractive index of the polymer waveguide with respect to wavelength $\lambda$, d is the distance between the top and bottom electrodes, and L is the length of the active area.

So the output of the proposed modulator shown in Fig. 2 is

$$I_{out} = \frac{1}{2}(I_1 + I_3 + I_5 + \ldots) - \frac{I_1}{2}\left[\cos(\Delta\Phi 1) + \frac{I_3}{I_1}\cos(\Delta\Phi 3) + \frac{I_5}{I_1}\cos(\Delta\Phi 5) + \ldots\right]. \tag{7}$$

Corresponding each cose($\theta$) item of Eq. (4) to that of Eq. (7), we can determine the active length of each Mach-Zehnder interferometer as following

$$\left\{\begin{array}{l} L_{MZ1} = \lambda d/(2n^3\gamma_{33}V_{max}) \\ \quad L_{MZ2} = 3L_{MZ1} \\ \quad L_{MZ3} = 5L_{MZ1} \\ \quad \quad \ldots\ldots \end{array}\right. \tag{8}$$

For an instance, the electro-optic coefficient of a typical polymer of 30 pm/v at wavelength 1.3 μm, and refractive index 1.55, assuming d=6.5μm and $V_{max}$=10V, the length of the Mach-Zehnders are 3.781mm, 11.346mm, 18.910mm, respectively.

The input optical power is distributed by the directional couplers according to the coefficients in front of the cose(θ) terms in Eq. (7). The coupling efficiency of each directional couplers should be

$$\left\{\begin{array}{l} \varepsilon_{DC1} = (1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots)^{-1} \\ \varepsilon_{DC2} = \frac{1}{3^2}(\frac{1}{3^2}+\frac{1}{5^2}+\ldots)^{-1} \\ \quad \varepsilon_{DC3} = \frac{1}{5^2}(\frac{1}{5^2}+\ldots)^{-1} \\ \quad \quad \ldots\ldots \end{array}\right. . \tag{9}$$

For a passive directional coupler with coupling coefficient κ and coupling length L, the output-input light power ratio is

$$\frac{I_{out}}{I_{in}} = \cos^2(\kappa L). \tag{10}$$

So the coupling length of each directional coupler should be

$$L_{DCi} = \frac{1}{\kappa}\arccos(\sqrt{\varepsilon_{DCi}}) \qquad (i=1, 3, 5,\ldots). \tag{11}$$

Assuming the refractive indices of the waveguide core material and cladding material at wavelength 1.3 μm are 1.5416 and 1.5409 respectively, the cross-section of the waveguide is 2.5μm×2.5μm, and the separation of the two waveguide is 9 μm, the coupling coefficient of the directional coupler can be calculated with coupled-wave equations as κ=0.300 rad/mm. For a linearized modulator that contains four Mach-Zehnders, the coupling efficiencies of the directional couplers are 0.8536, 0.6478, 0.6622, respectively, and the coupling length of each corresponding directional coupler are 1.3088 mm, 2.1179 mm , 2.0674 mm, respectively.

## 3. SPURIOUS SIGNAL ANALYSIS

The conventional method to characterize the linearity of a modulator, is to expand its transfer curve into Taylor series. With first three or five order approximation, the spurious signal is defined as the optical power ratio of the third harmonic signal to the base-frequency signal, and the intermodulation distortion in two-tone test method is defined as the optical power ratio of the intermodulation signal to one of the two closely-spaced equal-amplitude electrical modulating signals.[4,7] Both of these two methods are based on Taylor-series approximation, so that they can not precisely characterize a electro-optic modulator especially when the transfer curve of the modulator getting close to ideal. Because only the first severe orders of the expanded Taylor series are considered in these methods, while when the linearity of a modulator is improved close to ideal, high order items will have in-negligible contributions. This point has been predicted and also approved in our investigations. To precisely characterize the linearity of a modulator, we use Fourier- Spectrum-Analysis method. This method considers all of the high-order terms, and has no theoretical approximation. The procedures of this method is illustrated as following.

Fig. 4 shows the transfer curve of a modulator that contains only one Mach-Zehnder interferometer, which is actually the same as a conventional Mach-Zehnder interferometer with a optical DC bias. This modulator is driven by a sinusoidal electrical signal

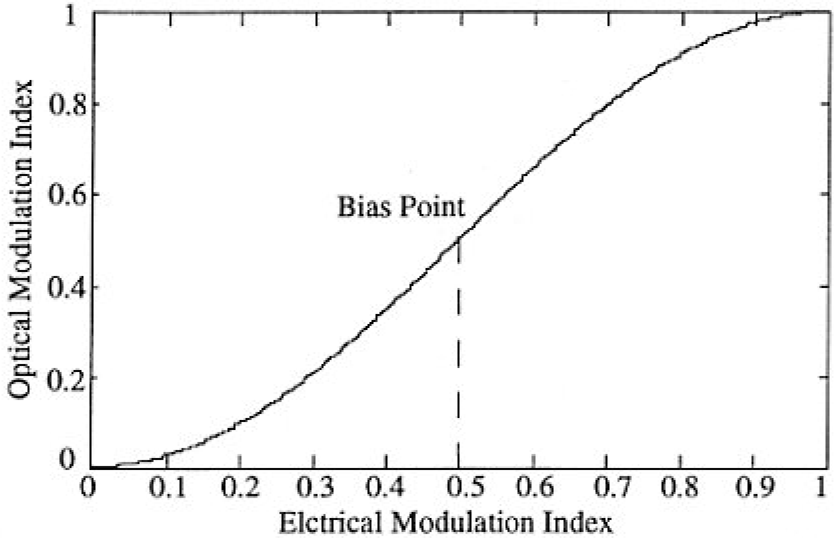

Fig. 4. Transfer curve of the linearized modulator that contain only one Mach-Zehnder interferometer.

$$V = V_{bias} + \frac{V_{max}}{2}\sin(2\pi f_0 t) \tag{12}$$

where $f_0$ is the frequency of the electrical modulation signal. The normalized optical output of the modulator is shown in Fig. 5, Fast Fourier transform gives out the power spectrum of this optical signal as shown in Fig. 6, which contain the basic frequency $f_0$ and the third harmonic frequency $3f_0$. The harmonic spurious signal is calculated as the relative power ratio of these two peaks. This harmonic spurious signal is independent of $f_0$.

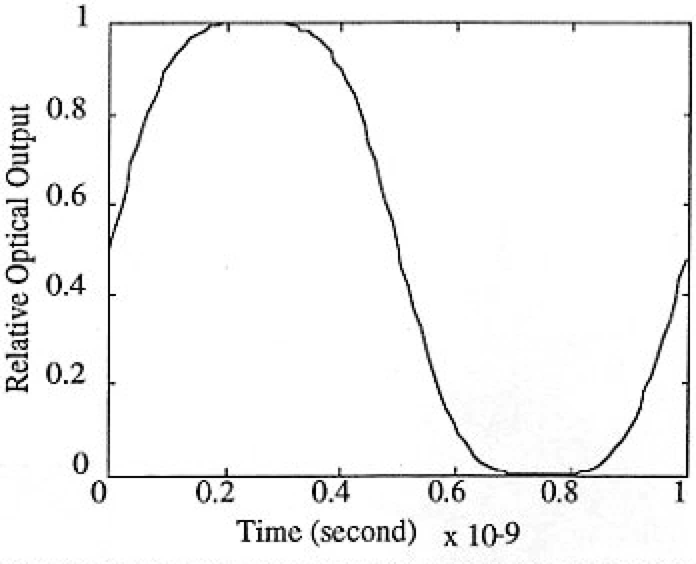

Fig. 5. Normalized optical output signal of the linearized modulator with optical modulation index 99%.

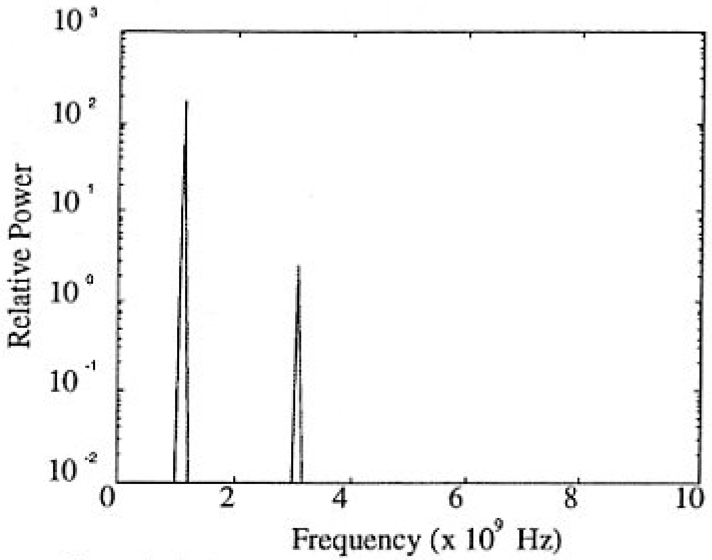

Fig.6. Relative power spectrum of the optical signal shown in Fig.5. $f_0$=1 Ghz.

Similarly, if this modulator is driven by a two-tone signal of frequency $f_1$ and $f_2$

$$V = V_{bias} + \frac{V_{max}}{b}\left[\sin(2\pi f_1 t) + \sin(2\pi f_2 t)\right] \tag{13}$$

where $b = 2\max\left[\sin(2\pi f_1 t) + \sin(2\pi f_2 t)\right] = 4$, (14)

the modulated output optical signal is shown in Fig. 7. Fast Fourier transform gives out the power spectrum of this signal as shown in Fig. 8, which include the two basic frequencies as well as the intermodulation spurious signal of $2f_1$-$f_2$ and $2f_2$-$f_1$. The Intermodulation distortion is then calculated as the relative power ratio of the intermodulation spurious signal to the basic signal.

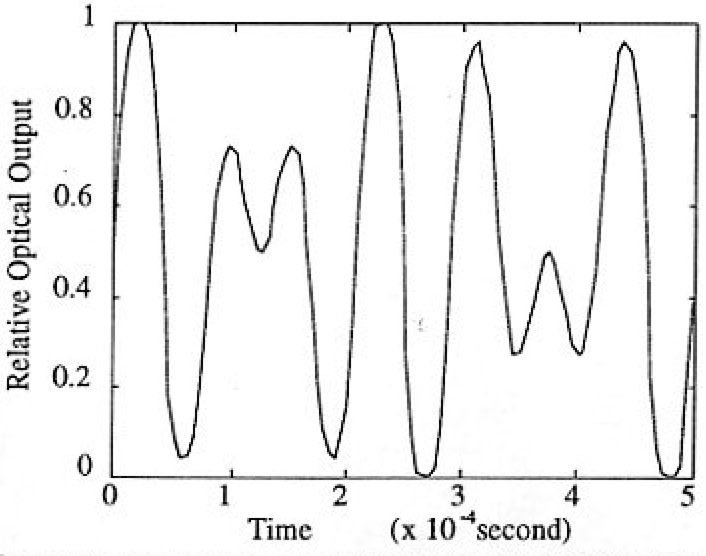

Fig. 7. Normalized optical output signal of the linearized modulator modulated by a two-tone electrical signal, $f_1$=10KHz, $f_2$=14KHz.

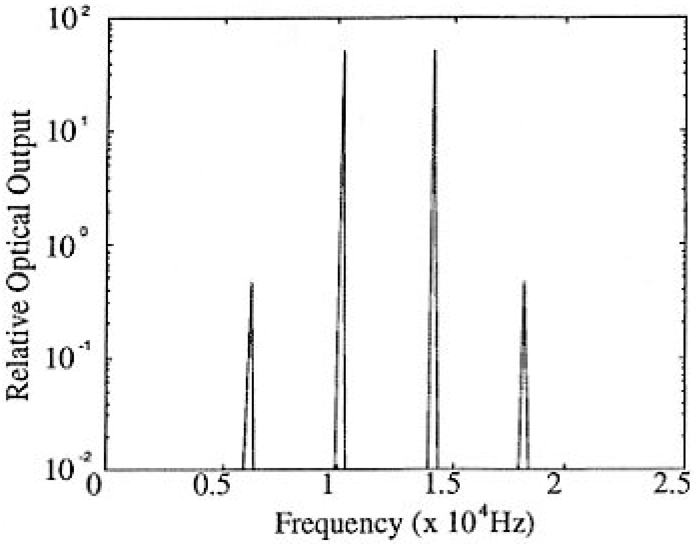

Fig. 8. Power spectrum of the output optical signal of the linearized modulator in two-tone test.

The transfer curves of linearized modulators composed of one to five Mach-Zehnders are shown in Fig. 9. We can see from Fig. 9 intuitively that the more Mach-Zehnders the modulators employ, the better the linearity.

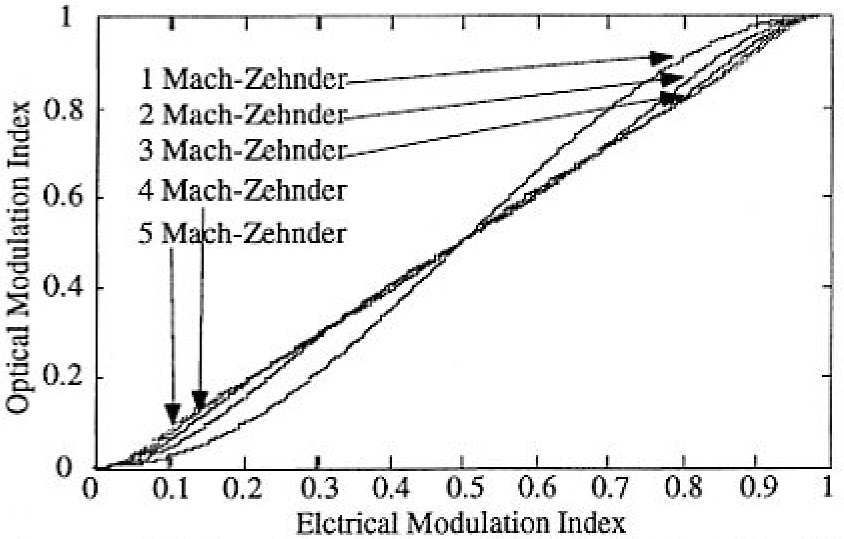

Fig. 9. Transfer curves of the linearized modulators composed of different numbers of Mach-Zehnder interferometers.

Fig. 10 shows the 3rd-harmonic spurious signal of this linearized modulator with different numbers of Mach-Zehnder interferometers employed. The third-intermodulation distortion of that is shown in Fig.11. Compare Fig.10 and Fig.11, one can see that the third-intermodulation distortion shows similar results as the third-harmonic spurious signal. Apparently, the more Mach-Zehnder interferometers a linearized modulator employs, the lower the spurious signal at high optical modulation index range. For a linearized modulator that contain two Mach-Zehnder interferometers, the spurious signal is higher than a conventional Mach-Zehnder interferometer when the optical modulation index is less than 80%, but is much lower when the optical modulation index is larger than 80%. That shows the contribution of the second Mach-Zehnder interferometer which canceled

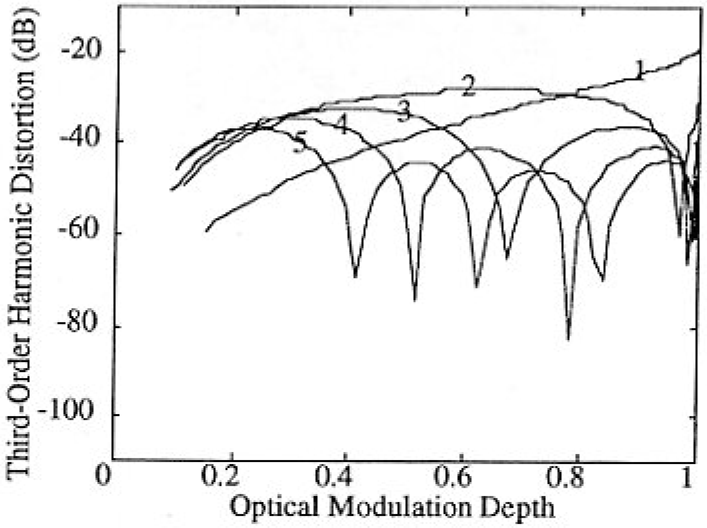

Fig. 10. Third harmonic distortion versus the optical modulation index of the linearized modulator composed of one to five Mach-Zehnder interferometers.

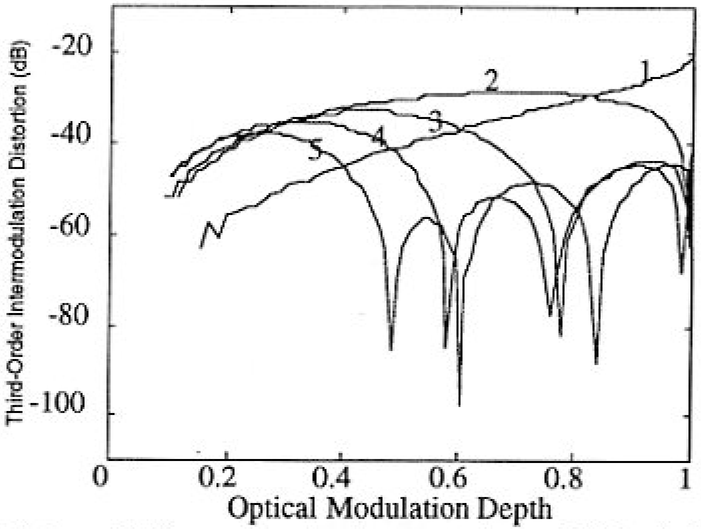

Fig. 11. Third-order-intermodulation spurious signal, versus optical modulation index, of the linearized modulator composed of one to five Mach-Zehnder interferometers.

the spurious signal to some extent when large optical modulation depth is used, and on the whole optical modulation range, the spurious signal keep the same order. Linearized modulators with three Mach-Zehnders can depress the spurious signal to -30dB on the whole optical modulation range. When four Mach-Zehnder interferometers are employed, the spurious signal is less than -40dB on the whole modulation range. A linearized modulator with five Mach-Zehnder interferometers shows even lower spurious signals, for example, less than -60 dB on the optical modulation index range from 40% to 65%. Further calculation shows theoretically that a linearized modulator with ten Mach-Zehnder interferometers, the spurious signal can be depressed as low as -60 dB on the whole optical modulation range. And one with 20 can achieve -70 dB.

## 4. CONCLUSION

A linearized external electro-optic modulator is designed with a set of Mach-Zehnder interferometers parallel cascaded via directional couplers. The linearity of the modulator can be modified by adjusting the numbers of Mach-Zehnder interferometers employed in the modulator depending on the request of concrete applications. Mach-Zehnder interferometers with different lengths cancel the spurious signal and result in a low-spurious-signal transfer curve on the whole optical modulation range, so that the spur-free optical modulation index can be as high as 100%, and the input optical power can be utilized efficiently. Input light power is distributed between Mach-Zehnder interferometers via passive directional couplers, so that only one input light signal is fed forward. The whole device is designed on a electro-optic polymer film which make it possible that the two arms of each Mach-Zehnder interferometer be inversely poled, and each Mach-Zehnder interferometer has a different length, so that the modulator need only one driving voltage, which guarantees the high-speed operation of the modulator. In the case of many Mach-Zehnder interferometers are used and the length of them may go beyond the practical feasibility, multiple driving voltages can be used as a trade-off to shorten the device length. As far as the fabrication tolerance of the length of the directional couplers is concerned, an adjustable DC electrical bias voltage for each directional coupler can be an option to compensate the fabrication error to achieve the expected coupling efficiency. Fourier-spectrum-analysis method is developed to characterize the modulator. This method is more precise than the conventional Taylor-series-approximation method. These theoretical results will serve as a good guide for the later practical experiments.

## 5. ACKNOWLEDGMENT

This research is sponsored by ONR, BMDO, and AFOSR.

## 6. REFERENCE

1. Moshe Nazarathy; Josef Berger; Ley, Levi Anthony J., Israel M. Levi; Yishai Kagan, , "Progress in externally modulated AM CATV transmission systems," Journal of Lightwave Technology, vol. 1, no. 1, pp 82-105, Jan 1993.
2. G.C.Wilson, T. H. Wood, J. L. Zyskind, J. W. Sulhoff, S. B. Krasulick, J. E. Johnson, T. Tanbun-Ek.; P. A. Morton, "Analogue transmission at 1.55 μm using linearized electroabsorption modulator/DFB laser and fiber amplifier," Electronics Letters, vol. 31, no. 22, pp 1934-1935, Oct 26 1995.
3. Robert Olshansky, "Optimal design of subcarrier multiplexed lightwave systems employing linearized external modulators," Journal of Lightwave Technology, vol.10, n0. 3, March 1992.
4. James H. Schaffner; William B. Bridges, "Intermodulation distortion in high dynamic range microwave fiber-optic links with linearized modulators," Journal of Lightwave Technology, vol. 11, no. 1, pp 3-6, Jan 1993.
5. C. K. Sun, S. A. Pappert, R. B. Welstand, J. T. Zhu, P. K. L. Yu, Y. Z. Liu and J. M. Chen, "High spurious free dynamic range fiber link using a semiconductor electroabsorption modulator," Electronics Letters, vol.31, no. 11, pp 902-903, May 1995.
6. Y. Trisno, L. K. Chen, and D. Huber, "A linearized external modulator for analog applications," SPIE vol.1371, pp 8-12, 1990.
7. William B. Bridges, James H. Schaffner, "Distortion in linearized electrooptic modulators," IEEE Transactions on Microwave Theory and Techniques, vol. 43, no.9, pp2184-2197, Sept 1995.
8. T. R. Halemane, and S.K.Korotky, "Distortion characteristics of optical directional coupler modulators," IEEE Transactions on Microwave Theory and Techniques, vol. 38, no.5, pp669-673, Sept 1990.
9. Gary E. Betts, L. M. Walpita, W. S. C. Chang, R. F. Mathis, "On the linear dynamic range of integrated electrooptical modulators," IEEE Journal of Quantum Electronics, vol. QE-22, no.7, pp1009-1011, July 1986.
10. Brian H. Kolner and David W. Dolfi, "Intermodulation distortion and compression in an integrated electrooptic modulator," Applied Optics, vol.26, no.17, pp3676-3680, Sept. 1987.
11. Catherine H. Bulmer and William K. Burns, "Linear interferometric modulators in Ti:LiNbO$_3$," Journal of Lightwave Technology, vol. LT-2, no. 4, pp512-521, August 1984.
12. Haeyang Chung, Willian S.C. Chang, and Eric L. Adler, "Modeling and optimization traveling-wave LiNbO$_3$ interferometeric modulators," IEEE Journal of Quantum Electronics, vol.27, no.3, pp608-617, March 1991.
13. Mark L. Farwell, William S.C. Chang, and David R.Huber, "Increased linear dynamic range by low biasing the Mach-Zehnder modulator," IEEE Photonics Technology Letters, vol.5, no.7, pp779-784, July 1993.
14. Mee Koy Chin, "Simple electrical biasing for minimizing the nonlinearities of quantum-well mach-zehnder modulators," Applied Optics, vol.33, no.9, pp1732-1734, Mar 20, 1994.
15. Steven K.Korotky, and Rene M. De Ridder, "Dual parallel modulation schemes for low-distortion analog optical transmission," IEEE Journal on Selected Areas in Communications, vol.8, no.7, pp1377-1380, Sept.1990.
16. Janet L.Brooks, Gregory S.Maurer, and Richard A.Becher, "Implementation and evaluation of a dual parallel linearization system for AM-SCM video transmission," Journal of Lightwave Technology, vol.11, no.1, pp34-41, January 1993.
17. James H. Schaffner, Willam B.Bridges, "A study of broadband microwave fiber optic links with linearized integrated-optic modulators," SPIE vol.2155,pp218-228, 1994.

18. Zong-qi Lin, and Williams S.C.Chang, "Waveguide modulators with extended linear dynamic range-A theoretical prediction," IEEE Photonics Technology Letters, vol.2, no.12, pp884-886, December 1990.

19. Pao-Lo Liu, B.J.Li, and Yudhi S. Trisno, " In search of a linear electrooptic amplitude modulator," IEEE Photonics Technology Letters, vol.3, no.2, pp144-146, February 1991.

20. Yu Wang-Boulic, "A linearized optical modulator for reducing third-order intermodulation distortion," Journal of Lightwave Technology, vol.10, no.8, pp1066-1070, August 1992.

21. Delfin Jay M. Sabido, IX, Masafumi Tabara, Thomas K.Fong, chung-Li Lu, and Leonid G.Kazovsky, "Improving the dynamic range of a coherent AM analog optical link using a cascaded linearized modulator," IEEE Photonics Technology Letters, vol.7, no.7, pp813-815, July 1995.

22. Juan F. Lam and Gregory L. Tangonan, "A novel optical modulator system with enhanced linearization properties," IEEE Transections Photonics Technology Letters, vol.3, no12, pp1102-1104, December 1991.

23. Kwang Tsai Koai and Pap-Lo Liu, " Digital and Quasi-Linear Electrooptic modulators synthesized from directional couplers," IEEE Journal of Quantum Electronics, vol. QE-22, no.12, pp2191-2194, December 1986.

24. James H. Schaffner, Juan F. Lam, Celestino J. Gaeta, Gregory L. Tangonan, Robert L. Joyce, Mark L.Farwell and William S.C.Chang, " Spur-free dynamic range measurements of a fiber optic line with traveling wave linearized directional coupler modulators," IEEE Photonics Technology Letters, vol.6, no.2, pp273-275, February 1994.

25. Mark L.Farwell, Zong-qi Lin, Ed wooten, and william S.C. Chang, "An electrooptic intensity modulator with improved linearity," IEEE Photonics Technology Letters, vol.3, no.9, pp273-275, September 1991.

26. Anders Djupsjobacha, "A linearization concept for integrated-optic modulators," IEEE Photonics Technology Letters, vol.4, no.8, pp869-872,August 1992.

William K. Burns, "Linearized optical modulator with fifth order correction," IEEE Journal of Lightwave Technology, vol.13, no.8, pp1724-1727, August 1995.