Implementation of De Bruijn and Kautz Bus Networks Using Waveguide Holograms

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ABSTRACT

In this paper, we discuss multihop optical bus interconnection networks. For massive parallel computing systems, such interconnection can interconnect a large number of processing elements and reduce system latency at the same time. We give the qualitative results of De Bruijn bus network and Kautz bus network as an engineering options based on combinatorial hypergraph theory and a uniform alphabetic constructive method. We also give an implementation method via polymer based waveguide holograms.

Keyword: Multihop Bus systems, De Bruijn bus networks, Kautz bus networks, Waveguide holograms.

1. INTRODUCTION

Interconnection network design problems become more and more important nowadays both in massive parallel computing systems and VLSI systems. When nodes in the system to be connected become large, this problem jeopardizes the interconnect performance at a low cost. The basic styles of interconnection are point to point interconnection and broadcast interconnection. In point to point communication, multihops may be needed to fully interconnect all the nodes in the system. In the broadcast interconnection, a simple way is to attach the processors all to a single bus. Other than these two methods, there is the third method-- multicast interconnection -- the grouped interconnection which communicate in point to point style between groups and in broadcast style within groups. One can image multicast as interconnected bus system.

There are many important parameters in designing an interconnect network. Their importance varies according to different implementation techniques. The most important parameters of a network are:

1) Diameter D: the maximum number of hops between any two nodes in the system. It represents maximum hops or times a message needed to go through any two nodes in system.

2) Degree Δ: the number of transmiter-receiver pairs each node has. The more the transmiters a node has, the more other nodes it can connect directly (the better of the connectivity of a network may has), but at more cost.

3) Volume r: the maximum number of nodes a bus can accommodate.

4) System size n: the total number of nodes a network can interconnect.

There are many other secondary parameters for a network. In this paper, we focus on these four parameters. The relationship among these parameters for a network design is delineated. The common goal for a high performance network is to have a network with a large system size n with optimized values of D, Δ, and r (possiblly small ones). However, in many cases, these parameters are contradictary. For

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1. INTRODUCTION

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There are many other secondary parameters for a network. In this paper, we focus on these four parameters. The relationship among these parameters for a network design is delineated. The common goal for a high performance network is to have a network with a large system size n with optimized values of D, Δ, and r (possiblly small ones). However, in many cases, these parameters are contradictary. For instance, with a fixed degree Δ, we need more hops (a larger D) or large bus volume r to reach more nodes. With a fixed diameter D(a fixed hops), we need more transmiter-receiver pairs Δ or larger bus volume r to reach more nodes. There are many options according to different network, but the maximum number of nodes $n(\Delta, D, r)$, for fixed Δ, D, and r, is limited by Moore bound:

$$n(\Delta,D,r) \le 1+\Delta(r-1)\Sigma_{i=0}^{D-1}(\Delta-1)^i(r-1)^i \quad \text{for uni-directional interconnects,}$$

$$n(\Delta,D,r) \le \Sigma_{i=0}^{D-1}(\Delta r)^i \quad \text{for bi-directional interconnects.}$$

where uni-directional interconnection means signals flow between any two nodes can only flow in one direction, like single optical fiber. Bi-directional interconnection means node A can send signals directly to node B, and node B can also send signals to node A. Uni-directional interconnections usually modeled as directed graphs in mathematical terminorlogy, while, bi-directional interconnections usually modeled as undirected graphs.

An interconnection network attain this Moore bound is known as Moore geometry[4][13]. There are many works aimed to search better network interconnect structure to accommodate more nodes. Some results are summarized in [4] [13]. Among them, de Bruijn network and Kautz network have better properties for system interconnection. They have been used in WDM[16], VLSI and computer systems interconnections[17].

We will use some mathematical tools to analyze networks. For instance, we use directed graphs as model for uni-directional interconnections, undirected graph as model for bi-directional interconnections, and hypergraph[8] as model for bus interconnections.

This paper is organized as follows. Section 2 discusses point to point interconnection networks using de Bruijn network and Kautz network. Section 3 delineates bus interconnection networks using de Bruijn and Kautz graphs. Section 4 presents more general cases by using the hypergraphic technology. And Section 5 comes with the concluding remarks.

2. DE BRUIJN NETWORK AND KAUTZ NETWORK

In this section, we discuss point to point interconnection using de Bruijn network and Kautz network. No bus is considered here. Before discussing de Bruijn and Kautz network, we first take a look at k-cube(or hypercube) which is the most popular point to point interconnection network. We can describe k-cube as nodes represented by binary sequences of length k(k tuple). Any two nodes are interconnected if and only if the two binary sequences have one bit difference. Such, a k-cube has diameter k with 2^k nodes each of which has only degree of k. If D=k, Δ=k, and by Moore bound, we have

$$n(\Delta,D) \le 1+\Delta(\Delta-1)+\cdots+\Delta(\Delta-1)^{D-1} = 1+k(k-1)+\cdots+k(k-1)^{k-1}$$
$$= \frac{k(k-1)^k-2}{k-2}$$

We can see from above that the number of nodes can be interconnected by k-cube(that is 2^k) are far from Moore bound when k is larger than 4, which means there may exist some better network structure. In fact, de Bruijn network and Kautz network are such network structures. We note that, if two networks a, b, with the same Δ and D, can interconnect n_a nodes and n_b nodes respectively, and $n_a >> n_b$, then we can conclude that , if we interconnect n_b nodes using network a, then it need only smaller number of average hops than using b to interconnect n_b nodes which makes system faster. Or network may need less degree for each node which means saving of transmitter-receiver pairs. Following table shows the comparison of nodes can be connected using different networks for some Δ and D, and their Moore bound.

n(Δ,D)	Δ = D = 4	Δ = D = 6	Δ = D = 8	Δ = D = 10
k-cube	16	64	256	1024
de Bruijn	16	729	65536	9765625
Kautz	24	972	81920	11718750
Moore bound	161	23437	7.7E+6	4.36E+9

Table 1 Number of nodes can be connected using different networks. Here de Bruijn and Kautz are undirected (see last part of this section)

As we can see, for a fixed Δ and D, the number of nodes can be accommodated in k-cube is less than that of de Bruijn network and that of Kautz network, although they all well bellow the theoretical limit-- Moore bound.

de Bruijn network

There are several ways to define de Bruijn network. These include words on an alphabet; line digraph iteration; and integer congruence. Here we use words on alphabet[21].

The de Bruijn network B(Δ,D) is a digraph (direct graph) that each vertice is a word of length D on an alphabet Σ of Δ letters. Two nodes u and v (that is, two words) will have an edge from u to v (denoted by $u \rightarrow v$) iff v is a left shifted version of u, that is, if $u = u_1u_2\cdots u_D$, then $u \rightarrow v$ exists iff $v = u_2u_3\cdots u_D\alpha$, for any $\alpha \in \Sigma$. Such, each node in de Bruijn network has out- degree Δ and in-degree Δ. Any two nodes in de Bruijn network need at most D hops to reach each other. We can show this easily as follows. Suppose $u = u_1u_2\cdots u_D$, $v = v_1v_2\cdots v_D$, then, $u_1u_2\cdots u_D \rightarrow u_2u_3\cdots u_Dv_1, \rightarrow u_3\cdots u_Dv_1v_2 \rightarrow \cdots \rightarrow v_1v_2\cdots v_D$, has D hops. Fig 1 shows a B(2, 3) network.

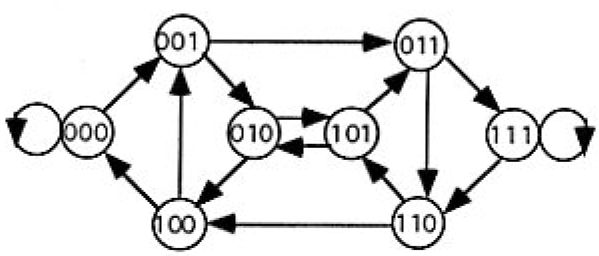

Fig. 1 De Bruijn network B(2, 3)

For a word of length D on an alphabet Σ having Δ elements, on each position in the word, we have Δ options, so totally we have Δ^D different words. Such, de Bruijn network B(Δ,D) has Δ^D nodes. And since de Bruijn network is a regular graph, each node has Δ out-going edges, it has totally $e = n \times \Delta = \Delta^{D+1}$ edges.

Kautz network

The Kautz network K(Δ,D) is a digraph that each vertice is a word of length D on an alphabet Σ of $\Delta+1$ letters, with a restriction that any two consecutive symbols in a word are different. Two nodes u and v (that is two words) will have an edge from u to v (denoted by $u \rightarrow v$) iff v is a left shifted version of u, that is, if $u = u_1u_2\cdots u_D$, then $u \rightarrow v$ exists iff $v = u_2u_3\cdots u_D\alpha$, for any $\alpha \neq u_D$, $\alpha \in \Sigma$. Such, each node in Kautz network has out- degree Δ and in-degree Δ. Any two nodes in Kautz network need at most D hops to reach each other. We can show this easily as follows. Suppose $u = u_1u_2\cdots u_D$, $v = v_1v_2\cdots v_D$, then, if

$v_1 \neq u_D$, $u_1u_2 \cdots u_D \rightarrow u_2u_3 \cdots u_Dv_1, \rightarrow u_3 \cdots u_Dv_1v_2 \rightarrow \cdots \rightarrow v_1v_2 \cdots v_D$, it has D hops. If $v_1 = u_D$, then $u_1u_2 \cdots u_D \rightarrow u_2 \cdots u_Dv_2 \rightarrow u_3 \cdots u_Dv_2v_3 \rightarrow \cdots \rightarrow u_Dv_2 \cdots v_D = v_1v_2 \cdots v_D$. Fig 2 shows a K(2, 2) and a K(2, 3) network.

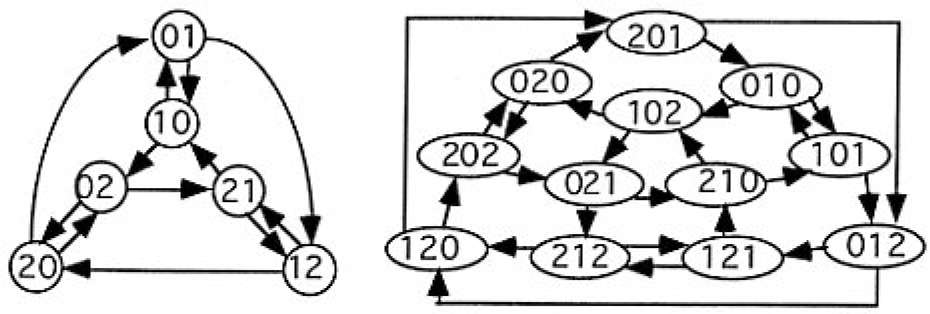

Fig. 2 Kautz Networks. Left: K(2, 2). Right : K(2, 3).

For a word of length D on an alphabet Σ having $\Delta+1$ elements, on the first position of the word we have $\Delta+1$ options, and on each of the rest positions in the word, we have Δ options. Totally, we have $(\Delta+1)\Delta^{D-1} = \Delta^D + \Delta^{D-1}$ different words. Such Kautz network $K(\Delta,D)$ has $\Delta^D + \Delta^{D-1}$ nodes. And since Kautz network is a regular graph, each node has Δ out-going edges, it has a total of total $e = n \times \Delta = \Delta^{D+1} + \Delta^D$ edges.

Undirected de Bruijn and Kautz networks

Undirected network is obtained from the directed one simply by replacing the directed edges with undirected edges, and remove possible loops and parallel edges. We denote an undirected de Bruijn network with an edge degree Δ and a diameter D as $UB(\Delta,D)$, and an undirected Kautz network $UK(\Delta,D)$. They can accommodate $(\Delta/2)^D + (\Delta/2)^{D-1}$ nodes for $UK(\Delta,D)$; $(\Delta/2)^D$ nodes for $UB(\Delta,D)$.

3. BUS NETWORK DESIGN WITH DE BRUIJN NETWORK

With the unique property of de Bruijn network, we will replace each edge of $UB(\Delta,D)$ with a bus. Such bus system has maximum diameter D+1. UB(2,3) and UK(2,2) are shown as follows, where each edge is replaced by a bus.

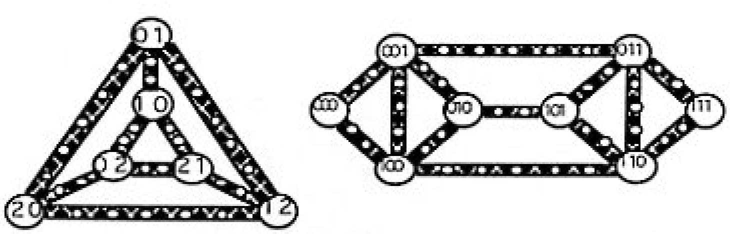

Fig. 3 on the left: UK(2,2) bus system. on the right: UB(2,3) bus system

In fig. 3, each white dot represents a fan-in/fan-out point where signal can be sent and received from other fan-in/fan-out point s along the same bus, including the end numbered end points which are also fan-in/fan-out points. For example, any white dot on the bus between 01 and 20 can reach 02 in two hops; any white dot on the bus between 000 and 100 can reach 111 in three hops. Here, each bus is a polymer based holographic waveguide.

Holographic waveguide is an optical device to provide signal routing. A laser beam injected into holographic waveguide diffracted at the holographic layer according to the Brag diffraction condition. Fig. 4 shows the diffraction diagram for fan-in. On the left, K_{in} is the fan-in beam vector, $|K_{in}| = 2\pi n / \lambda$, where n is the refractive index, λ is the wavelength of the laser beam. K^1_{out} is the diffracted beam vector within waveguiding plate. K_1 is the needed grating vector to get the resulting vector, which is perpendicular to the grating orientation. $|K_1| = 2\pi / \Lambda$, where Λ is the period of the grating. Our objective is to make a hologram having the pattern of K_1 to redirect the laser beam to the left with a bouncing angle of 45 degree. The right side of Fig. 4 shows another grating pattern (K_2, perpendicular to the grating orientation) which redirects the laser beam to the right in 45 degree. K_{undi} represents the undiffracted beam. Fig. 5 shows diffraction diagram for fan-out. Since our design is based on 45 degree principle, surface-normal fan-in results in several fan-our that also are surface-normal according to properly grating K_1 and K_2 as shown in the vector diagram in fig. 5.

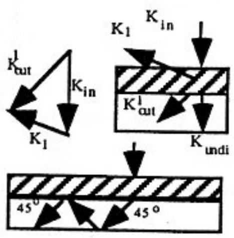

Fig. 4 Diffraction diagram for fan-in

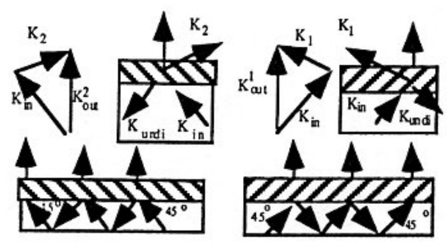

Fig. 5 Diffraction diagram for fan-out

Fig. 6 shows a multiplexed holographic grating. Two exposures applied to one light- sensitive polymer attached to substrate. When one processor sends a light beam signal, any other processor receives the fan-out signal. Multiplexed vector diagrams are also shown in fig. 6.

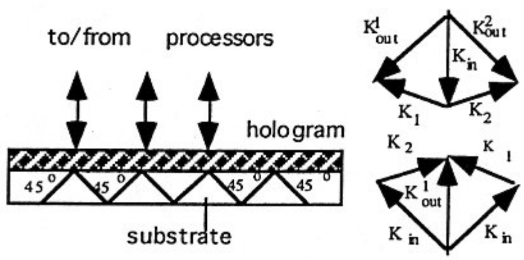

Fig. 6 Multiplexed polymer based hologram waveguide.

Fig. 7 shows our setup for recording the hologram. The laser beam from an Argon laser is splitted into two beams via a beam splitter. One is the object beam, the other is the reference beam. If we denote the angle between these two beams as θ, then the grating period Λ is $\Lambda = \lambda / \{2n_r \sin(\theta / 2)\}$, where λ is the wavelength and n_r is the refractive index of the air.

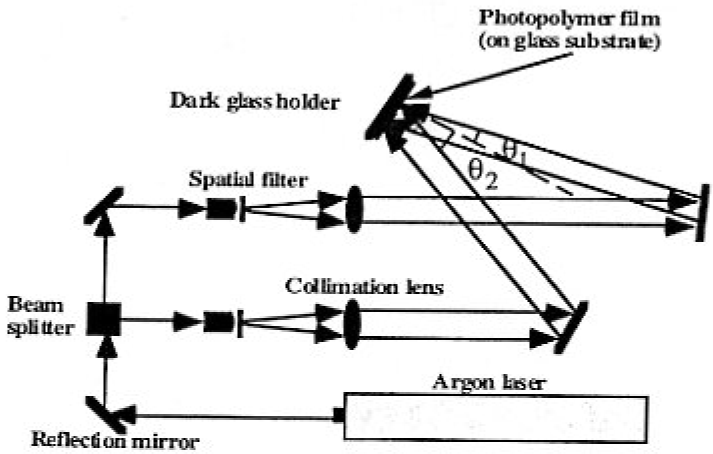

Fig. 7 Setup for recording hologram.

The reconstruction wavelengths may be different from the recording wavelength. Such, to form a slanted grating coupler which converts a vertical incident wave into a total internal reflection substrate guided mode with diffraction angle α (there is 45 degree), the two incident angles of the recording beams are known as:

$$\theta_1 = \sin^{-1}\left\{\frac{n}{n_r}\sin\left[\frac{\alpha}{2}+\sin^{-1}\left(\frac{\lambda_b}{\lambda_r}\sin\frac{\alpha}{2}\right)\right]\right\},$$

$$\theta_2 = \sin^{-1}\left\{\frac{n}{n_r}\sin\left[\frac{\alpha}{2}-\sin^{-1}\left(\frac{\lambda_b}{\lambda_r}\sin\frac{\alpha}{2}\right)\right]\right\}.$$

where n is the reflective index of the hologram, λ_b and λ_r represent the wavelengths of the recording and the reconstruction waves, respectively.

On the right of fig. 7, there shows the resulting multiplexed hologram waveguide bus. By replacing each edge of UB(2,3) with such bus, we get Fig. 8.

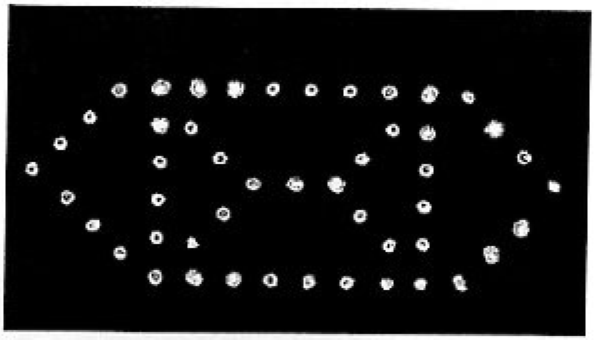

Fig. 8 Realization of UB(2,3) using waveguide holograms

If maximal r nodes can be attached to each bus, then, for de Bruijn B(Δ ,D), the total number of nodes is $\Delta^D(r\Delta-2\Delta+1)$. And for Kautz K(Δ ,D), the total number of nodes can be accommodate is $(\Delta^D+\Delta^{D-1})(r\Delta-2\Delta+1)$. Note that, the buses here are bidirectional. If they are unidirectional, the results hold.

In the next section, we will see that bus network can accommodate more nodes with fixed parameters of Δ, D, r by using hypergraph..

4. BUS NETWORK DESIGN WITH HYPERGRAPH

Hypergraph

A graph is a set of nodes connected by edges. Each edge connected only two nodes. Hypergraph extend a graph in that each edge can connect more than two nodes, such edge called hyperedge(for more mathematical definitions, see[8]). An n-node bus system is a hypergraph with n nodes and one hyperedge. A hypergraph is denoted as (Δ, D, r), where r is the volume of the hyperedge(maximum nodes can be accommodated within an hyperedge), Δ , and D as defined before. The maximum number of nodes a network may have is limited by Moore bound:

$$n(\Delta, D, r) \le 1+\Delta(r-1)\Sigma_{i=0}^{D-1}(\Delta-1)^i(r-1)^i.$$

Dual Hypergraph

Given a hypergraph H, we can take its hyperedge as another hypergraph's node(the volume r of the hyperedge becomes the degree Δ of nodes in the new hypergraph), its node as the hyperedge of the new hypergraph(node degree Δ becomes the bus volume r of the new hypergraph. We will call this new hypergraph the dual hypergraph of original one, denoted as H^*. It is well known [13] that, if H is a (Δ, D, r)-hypergraph, then its dual hypergraph is a (r, D^*, Δ)-hypergraph , where D-1 $\le D^* \le$ D+1. This means the difference of diameters of a hypergraph and its dual is at most one. It's interesting to construct hypergraph (2, D, Δ) from the ordinary graph (Δ, D, 2) (that is the regular graph (Δ, D)). For ordinary de Bruijn network(or Kautz network) with node degree Δ and diameter D(note that regular graph has bus volume r=2), by using hypergraph dual operation, we get a hypergraph with bus volume Δ and each node on two buses.

De Bruijn Bus Network and Kautz Bus Network

De Bruijn Bus Network

Let's denote de Bruijn bus network as B(Δ, D, k) where Δ is the degree of each node, D is the maximum hop between any two nodes in the network, k is the number of nodes can be connected to a bus. Here, we discuss two different cases. First, we consider the case where each node's transmitters and receivers may attach to different buses, each bus can accommodate k transmitters and k receivers from different nodes, as denoted as B(Δ, D, k). In the second case, the transmitter and receiver of each nodes are unseparable(i.e. transmitter-receiver pair should be attached to same bus), the maximum number of nodes a bus can accommodate is k, we denoted such de Bruijn network as UB(Δ, D, k).

Since the representation of bus network can be done using graph, we need hypergraph representation where each edge is a hyperedge which means there may be more than 2 nodes on an edge. Each hyperedge is a bus. If there are k nodes are incident to the hyperedge, then it means there are k transmitters of different nodes are attached to the bus. If there are k nodes are incident from the hyperedge, then it means there are k receivers attached to the bus.

De Bruijn bus network $B(\Delta, D, k)$ are constructed as follows. We code each node as an element $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$, where $w_i \in \Sigma_k$, Σ_k is an alphabet with k elements, and $u_i \in \Sigma_\Delta$, Σ_Δ is a alphabet with Δ elements. We also code each bus as $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}\beta$, where $w_i \in \Sigma_k$, $u_i \in \Sigma_\Delta$, and $\alpha,\beta \in \Sigma_\Delta$. The length of $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ is equal to the length of $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}\beta$. They both are equal to D. For each node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$, it incident to the set of hyperedges $u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\alpha$, where $\alpha \in \Sigma_\Delta$. That is, there is an arc pointing to the hyperedge $u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\alpha$ from the node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$. Since there are Δ elements in Σ_Δ, each nodes incident to Δ different hyperedges. We should keep in mind that hyperedge itself is a bus. So, each node incident to Δ different buses. The node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ is also incident from the set of hyperedges $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}u_D$, where $\alpha \in \Sigma_\Delta$. That is, there is an arc pointing to the node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ from hyperedge $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}u_D$. Since there are Δ elements in Σ_Δ, there are Δ hyperedges connect to (or point to) the node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$. The in-degree of each node is Δ, and the out-degree of each node is Δ. We can think this situation as there is a node with Δ transmitters and Δ receivers, which transmits signal to Δ buses and receives signal from Δ buses simultaneously. As to each bus, from $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D \rightarrow u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\alpha$ we know that there are k transmitters attached to the bus, since $w_1 \in \Sigma_k$ and Σ_k has k elements. From $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}u_D \rightarrow w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$, there are also k receivers attached to the bus, since $w_D \in \Sigma_k$ and Σ_k has k elements.

With such an interconnected de Bruijn Network, the maximum hops needed from any node in the network to any other node (that is the diameter of the hypergraph) is D. We can simply show this as follows. In order to hop from node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ to node $\gamma_1\lambda_1\gamma_2\lambda_2\cdots\gamma_D\lambda_D$, we have

$$\begin{aligned} w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D &\rightarrow u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\lambda_1 \\ &\rightarrow w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\gamma_1\lambda_1 \\ &\rightarrow u_2\cdots w_{D-1}u_{D-1}w_Du_D\gamma_1\lambda_1\lambda_2 \\ &\rightarrow \cdots \\ &\rightarrow \gamma_1\lambda_1\gamma_2\lambda_2\cdots\gamma_D\lambda_D. \end{aligned}$$

There are at most 2D shifts, which visits at most D hyperedges (D hops).

The maximum number of nodes in a de Bruijn network can be calculated as follows. Each $w_i \in \Sigma_k$ in $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ takes k different values and each $u_i \in \Sigma_\Delta$ takes Δ different values. The total possibilities will be $(\Delta k)^D$. So, $n=(\Delta k)^D$.

For $UB(\Delta, D, k)$ (that is a network with Δ T/R pairs, maximum D hops and bus volume k), we should construct the de Bruijn network $B(\Delta/2, D, k/2)$ and replace each directed arc with undirected arc. Such the maximum number of nodes it can interconnect is $\left(\frac{\Delta k}{4}\right)^D$.

Kautz bus network

Kautz bus network $K(\Delta, D, k)$ are constructed as follows. Similar to de Bruijn network, we code each node as an element $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$. It consists of D w_iu_i pairs where $w_i \in \Sigma_k$ (Σ_k is an alphabet with k elements), and $u_i \in \Sigma_\Delta$ (Σ_Δ is an alphabet with Δ elements) with the restriction that any two consecutive w_iu_i pairs are different. Although each w_iu_i pair has $k\Delta$ options, we add a default state **(i.e., w_iu_i=**, for some i), such, each w_iu_i has $k\Delta+1$ options. We also code each bus as $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}\beta$, where $w_i \in \Sigma_k$, $u_i \in \Sigma_\Delta$, and $\alpha, \beta \in \Sigma_\Delta$. For each node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$, it incident to the set of hyperedges $u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\alpha$, according to that, 1) if w_Du_D=**, then $\alpha \in \Sigma_\Delta$ could be any element in Σ_Δ, 2) if $w_Du_D \neq$**, then $\alpha \neq u_D$, it may take any element in $(\Sigma_\Delta - u_D) \cup (*)$. Since there are Δ elements in Σ_Δ, each nodes incident to Δ different hyperedges. The node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ is also incident from the set of hyperedges $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}u_D$, according to that, 1) if u_D= *, then w_D= *, 2) if $u_D \in \Sigma_\Delta$, then $w_D \in \Sigma_k$ ($w_D \neq *$). That is, there is an arc point to node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ from hyperedge $\alpha w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}u_D$. Since there are Δ elements in Σ_Δ, there are Δ hyperedges connect to (or point to) the node $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$. The in-degree of each node is Δ, and the out-degree of each node is Δ. As to each hyperedge, from $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D \to u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\alpha$, we know that there are at most k transmitters attached to the bus, since $w_1 \in \Sigma_k$ and Σ_k has k elements, if $u_1 \neq *$. If u_1= *, then only one node is attached to the bus. There are also at most k receivers attached to the bus. Note that, if u_D= *, then w_D= *, and only one receiver is attached to the bus.

With such an interconnected Kautz Network, the maximum hops needed from any node in the network to any other node is D. We can simply show this as follows. From $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ to node $\gamma_1\lambda_1\gamma_2\lambda_2\cdots\gamma_D\lambda_D$, if $w_Du_D \neq \gamma_1\lambda_1$ we have

$$\begin{aligned} w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D &\to u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\lambda_1 \\ &\to w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D\gamma_1\lambda_1 \\ &\to u_2\cdots w_{D-1}u_{D-1}w_Du_D\gamma_1\lambda_1\lambda_2 \\ &\to \cdots \to \gamma_1\lambda_1\gamma_2\lambda_2\cdots\gamma_D\lambda_D. \end{aligned}$$

There are at most 2D shifts, which visit at most D hyperedges (D hops).

If $w_Du_D = \gamma_1\lambda_1$, then $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D = w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}\gamma_1\lambda_1 \to u_1w_2u_2\cdots w_{D-1}u_{D-1}\gamma_1\lambda_1\lambda_2 \to w_2u_2\cdots w_{D-1}u_{D-1}\gamma_1\lambda_1\gamma_2\lambda_2 \to \cdots \to \gamma_1\lambda_1\gamma_2\lambda_2\cdots\gamma_D\lambda_D$, there are at most 2D shifts, that is, at most D hops.

The maximum number of nodes in a Kautz bus network can be calculated as follows. Each $w_iu_i \in (\Sigma_k \times \Sigma_\Delta) \cup (**)$ in $w_1u_1w_2u_2\cdots w_{D-1}u_{D-1}w_Du_D$ takes $\Delta k+1$ different values, and any two consecutive $w_{i-1}u_{i-1}$, w_iu_i are different. So, the first place of pair w_1u_1 may takes $\Delta k+1$

possibilities, and the rest each pair may takes Δk possibilities. The total possibilities will be $(\Delta k+1)(\Delta k)^{D-1} = (\Delta k)^D + (\Delta k)^{D-1}$. So, $n = (\Delta k)^D + (\Delta k)^{D-1}$.

For UK(Δ, D, k) (that is a network with Δ T/R pairs, maximum D hops and bus volume k), we should construct the Kautz network K(Δ/2, D, k/2) and replace each directed arc with undirected arc. Such the maximum number of nodes it can interconnect is $\left(\frac{\Delta k}{4}\right)^D + \left(\frac{\Delta k}{4}\right)^{D-1}$.

Bus network: dual between bus volume and T/R pairs

Duality between bus volume r and node degree Δ: If bus volume r and nodes degree Δ are interchangeable to connect a same number of nodes, we say the network has dual property between bus volume and nodes degree. From $n=\left(\frac{\Delta r}{4}\right)^D$, we know de Bruijn network has dual property. And from $n=\left(\frac{\Delta r}{4}\right)^D + \left(\frac{\Delta r}{4}\right)^{D-1}$, Kautz network also has dual property. If network has duality property, then $n(\Delta, D, r)=n(r, D, \Delta)$. On the other hand, since $n(\Delta, D, r) = n(r, D^*, \Delta)$, we have $n(r, D, \Delta)=n(r, D^*, \Delta)$ and $D^*=D$.

In point to point communication as we discussed in section 2, one need large Δ (large number of transmitter-receiver pairs) in order to connect larger network with the same D(delay). According to dual property, Δ and r are interchangeable, we can put buses in point to point network and reduce Δ, while D (delay) keep unchanged. Bus save transmitter-receiver pairs!

5. CONCLUSION

In this paper, we show how to construct a good interconnection network and how it can be used in designing bus networks. De Bruijn network and Kautz network provide better options for implementation. For a given number of transmitter-receiver pairs Δ, system diameter(delay) D and the maximal number of nodes r that a bus can accommodate, the total number of nodes de Bruijn network can accommodate is $\left(\frac{\Delta r}{4}\right)^D$, Kautz network is $\left(\frac{\Delta r}{4}\right)^D + \left(\frac{\Delta r}{4}\right)^{D-1}$. If we permit one node's transmitter and receiver on different buses, then the number of nodes de Bruijn network can accommodate is $(\Delta r)^D$, Kautz network is $(\Delta r)^D + (\Delta r)^{D-1}$. The number of nodes we can connect when transmitter and receiver are separable is larger than the restricted case where the transmitter and receiver of one nodes must be on the same bus.

According to the dual property, Δ and r are interchangeable. So the significance of the bus network is, we can put buses in point to point network and reduce Δ, while D (delay) keep unchanged. Bus save transmitter-receiver pairs!

Although de Bruijn and Kautz network give a constructive way to build networks with clear parameters options, they are not optimal. There exist some networks even better than de Bruijn network or Kautz network. But due to the difficulties of constructive method, there still need a lot of efforts to find out networks near Moore bound.

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