

## Nonlinear optical processing using phase grating

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### ABSTRACT

Intensity-to-phase modulation can be used for phase coding of an input image, instead of the commonly employed halftone technique. When placed at the entrance of a  $2f$  coherent system with the selected diffraction order, the phase-coded image shows nonlinear behavior. No special screen design is necessary and a simple Ronchi-type or sinusoidal gratings can be used. Using DCG, we attained an input dynamic range of three order of magnitude, a significant improvement over previous work. A multipeak transfer function was obtained by photoresist as a recording media, which is useful for equidensitometry. Generally, the phase-coding for the nonlinear optical processor is more light efficient and allows a real time implementation with an erasable photopolymer material.

### 1. NONLINEAR TRANSFORMATIONS WITH HALFTONES

Coherent optical systems are employed for all-optical image processing. Theoretically, they are able to perform a variety of linear transformations, with the linearity being an inherent property of the coherent systems. Besides nonlinear material and devices such as optically addressed spatial light modulators, the only known way to perform a nonlinear transformation was to halftone the input image. The halftone process originated with the graphic arts business and was based on the high contrast registration of the original image through a grid.

It happened that the same scientist, namely W. H. Fox Talbot, was the first to discover both a self imaging diffraction phenomena (Talbot effect [1]) and a halftone process for printing [2]. Since that time (1836 and 1852, respectively for the two discoveries), progress in both has taken divergent paths. Recent advances in the study of diffraction patterns of plane periodic objects did not affect the mostly intuitive way the printing industry does halftone negatives. The introduction of high resolution scanners further delayed solving the problems of classical halftoning, and the majority of printing plants still use a camera with a crossline screen to halftone large-format graphics, an expedient method.

The halftone process was again brought to the attention of the optical community in an unexpected manner, when Marquet and Tsujiuchi [3], and later Goodman and Kato [4] in this country, pointed out that a halftone image can be used for an all-optical nonlinear transformation. This nonlinear transformation of a picture's gray levels can be accomplished in a  $2-F$  coherent linear system by simply selecting one diffraction order in a Fourier plane. The only complication is that a halftoned original must be used as an input image.

In order to perform a halftoning, or in other words an intensity-to-space modulation, a process of contact copying has been chosen by all researchers so far [5-10]. This, of course, is the easiest way to make a halftone image, but the problem of halftone screen design and fabrication immediately arises. To design a halftone screen means to determine a 2-D (1-D in some cases) screen cell transmittance profile. Although the method of a halftone screen design for contact copying is well

developed, the fabrication of such a screen relies basically on a costly computer controlled microdensitometer, since a desired transmittance profile has to be formed within one cell, typically of 50-100  $\mu\text{m}$  dimension. This quite time consuming fabrication excludes any possibility of real time implementation, which is an ultimate goal for optical nonlinear processors.

A halftone process can be described by a one-dimensional analysis. After hard-clipping type of recording, we obtain a set of rectangular transmittance gratings having the same period  $T$ , but with a duty cycle ratio that is a function  $q(I_{in})$  of input intensity  $I_{in}$ . The local transmittance of the halftoned image is

$$t(x) = 1 - \sum_{n=-\infty}^{\infty} \text{rect}\left(\frac{x - nT}{qT}\right) \quad (1)$$

The intensity distribution in the Fourier plane is expressed by

$$T(v) = F\{t(x)\} = \delta(v) - \sum_{n=-\infty}^{\infty} \delta\left(v - \frac{n}{T}\right) Tq \text{Sinc}(\pi v Tq) \quad (2)$$

where  $\delta(v)$  is the Kronecker delta function.

Thus, if the  $n$ -th diffraction order is selected, the transformation rule from input intensity  $I_{in}$  to output intensity  $I_{out}$  is

$$I_{out} = \frac{T^2}{\pi^2 n^2} \sin^2[\pi n q(I_{in})] \quad (3)$$

Here, the function  $q$  is determined by the recording material characteristic curve and the halftone screen transmittance profile.

This procedure can be described in image processing terms. Any diffraction order selected in the Fourier plane of the conventional two-lens (2f) coherent optical processor contains the full input image information. This is a consequence of the modulation of the input image by a periodic structure with spatial frequency higher than the maximum image spatial frequency. (In fact, some losses in resolution of the transformed image are always assumed.) This simple modulation, however, does not lead to image transformation through filtering in the Fourier plane. There must also be an encoding the input-image gray levels by the intensity of the spectral components, or in other words, a mapping of the input gray levels to a distribution on the Fourier plane.

## 2. INTENSITY-TO-PHASE MODULATION

Next we propose a type of coding where the local input intensity affects another grating parameter such as amplitude. Clearly, a simple recording on transmittance-function-type material (silver halide negative film, etc.) would not produce a nonlinear, nonmonotonic transformation. A phase-modulating material, however, will nonmonotonically record the intensity in a given diffraction order.

The experimental feasibility of such a nonlinear processor was demonstrated by using dichromated gelatin (DCG), photoresist and erasable PVA photopolymers.

The nonlinear, nonmonotonic nature of the phase recording becomes clear from the expression for Fraunhofer diffraction on a purely phase sinusoidal grating specified by the transmittance function:

$$t(x) = \exp\left\{j\alpha \sin\left(2\pi \frac{x}{T}\right)\right\} \quad (4)$$

From a well known expansion we have:

$$\exp\left\{j\alpha \sin\left(2\pi \frac{x}{T}\right)\right\} = \sum_{n=-\infty}^{\infty} J_n(\alpha) \exp\left\{j2\pi n \frac{x}{T}\right\} \quad (5)$$

it follows that the intensity of the  $n$ -th selected Fourier component of a diffraction spectrum will be proportional to  $J_n^2(\alpha)$ , where  $J_n(\alpha)$  is the  $n$ -th order Bessel function. This means that :

$$I_{out} \sim J_n^2[\alpha(I_{in})] \quad (6)$$

since the recorded modulation is a function of the input intensity.

From this we see that phase coding provides several advantages over conventional halftoning. Even the zero diffraction order, which is the most effective for transferring information through the processor, has nonmonotonic behavior. Generally, a coherent processor with phase transparency input is more light effective than with a halftoned input image. As a result of phase transparency input, the high diffraction orders are easily observable. Moreover, we do not have a strong restriction on the grating period, since we use a simple grating instead of designing a special screen.

Eq. (6) is an approximate description of the diffraction on a phase grating. It is only valid for the so called "thin" grating<sup>(11)</sup>. The examples of diffraction efficiency for different types of gratings

are shown in Table 1 where  $\alpha = \frac{\pi d \Delta n}{\lambda \cos \theta}$  is a phase amplitude factor defined through index modulation  $\Delta n$ , film thickness  $d$  and angle of incidence  $\theta$ . According to the table a number of nonlinear transformations can be produced using simple types of gratings.

Table 1

$\tau(x) = \exp \left[ -j \frac{2\pi n d f(x)}{\lambda \cos \theta} \right]$		
Grating shape $f(x)$	Diffraction efficiency $\eta_m$	
	"zero" order	$\pm 1^{\text{st}}$ order
Sinusoidal $\sin 2\pi \frac{x}{\Lambda}$	$J_0^2 \alpha$	$J_1^2 \alpha$
Square $\begin{cases} 1 & \text{for }  x  \leq \frac{\Lambda}{2} + \Lambda_n \\ 0 & \text{for }  x  > \frac{\Lambda}{2} + \Lambda_n \end{cases}$	$\cos^2 \left( \frac{\pi \alpha}{2} \right)$	$\frac{4}{\pi^2} \sin^2 \left( \frac{\pi \alpha}{2} \right)$
Triangle $1 - 2 \left  \frac{x}{\Lambda} - h \right  \text{ for }  x  \leq \frac{\Lambda}{2} + \Lambda_n$	$\frac{\sin^2 \left( \frac{\pi^2 \alpha}{4} \right)}{\frac{\pi^2 \alpha}{4}}$	$\frac{\sin^2 \left( \frac{\pi^2 \alpha}{4} \pm 1 \right)}{\left( \frac{\pi^2 \alpha}{4} \pm 1 \right)^2}$
Sawtooth $1 - 2 \frac{x}{\Lambda} - h \text{ for } x \neq \Lambda_n$ $0 \text{ for } x = \Lambda_n$	$\frac{\sin^2 (\pi \alpha)}{\pi^2 \alpha^2}$	$\frac{\sin^2 (\pi \alpha)}{\pi^2 (\alpha \pm 1)^2}$

### 3. PROOF-OF-CONCEPT EXPERIMENTAL RESULTS

A conventional 2-f coherent optical processor was set up as shown in Figure 1, and the output intensity profile was registered by a CCD video camera connected with a PC based image acquisition system. Any horizontal line of an intensity profile could be displayed and printed out. This system examined a specially prepared mask that simulated a halftoned image with a set of gratings with the same period. The duty cycle rotation, however, varies linearly, from 0.03 to 0.97. The transfer functions in Figure 2 show agreement with the predictions based on Eq. (3). This is a supportive factor for the planned processor prototype. We estimated the mean squared error to be less than 10% and the intensity fluctuation is mainly due to the ground glass placed in the observation plane.

To make observation more clear, we used a line averaging technique to obtain an ensemble mean. The ensemble averaged intensity for the same output signals is shown in Figure 2.

In the phase coding experiment, two types of photosensitive materials were used: DCG and photoresist. When a sinusoidal grating ( $T = 50$  mm) was copied onto the DCG film, the optical index modulation of the film was the prevailing factor. The object was simulated by a continuous neutral density filter with optical density (OD) changes from almost 0 to 3.0. After standard

development, the plate was placed at the entrance of the processor. An example of the transfer function for the selected first diffraction order is shown in Figure 3. A transformation dynamic range of three orders of magnitude was observed, which is a significant improvement over previous work. We consider this enlarged dynamic range to be an important advantage of phase coding.

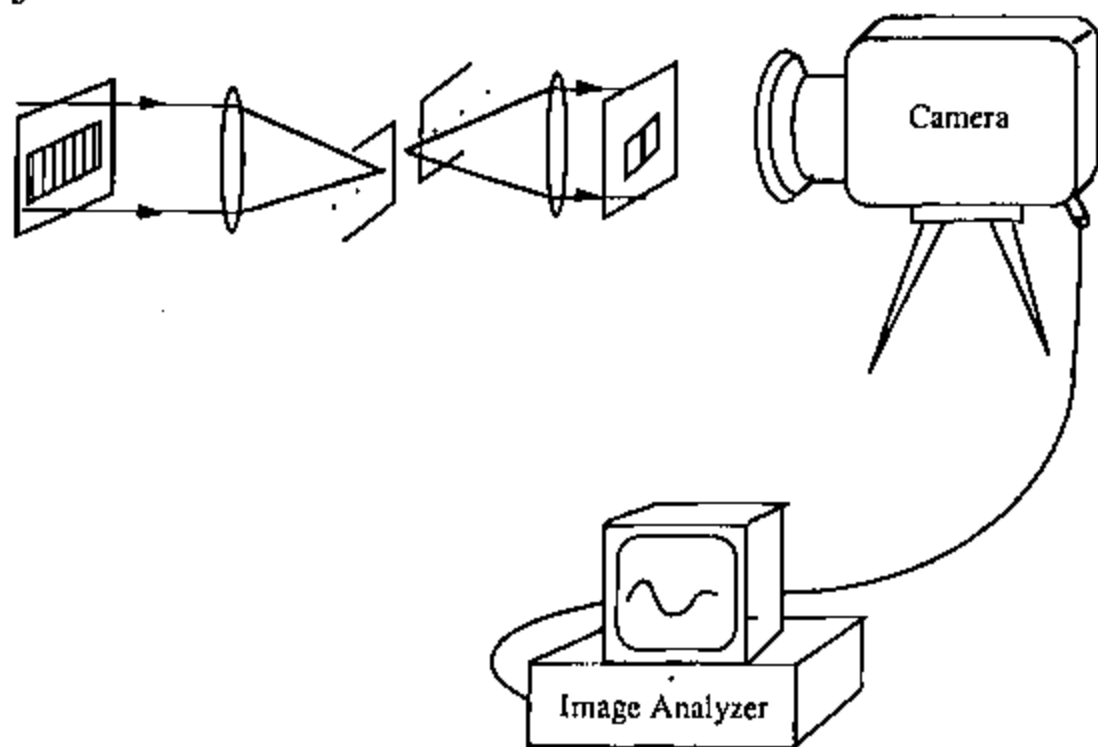
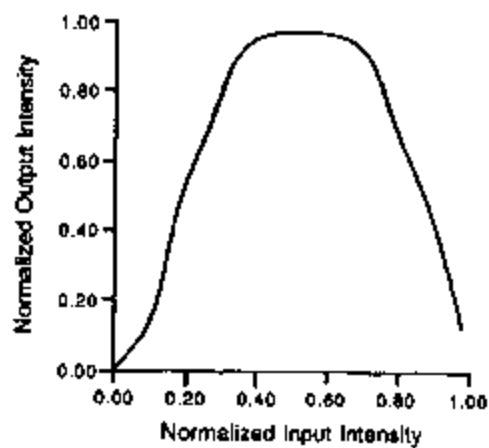
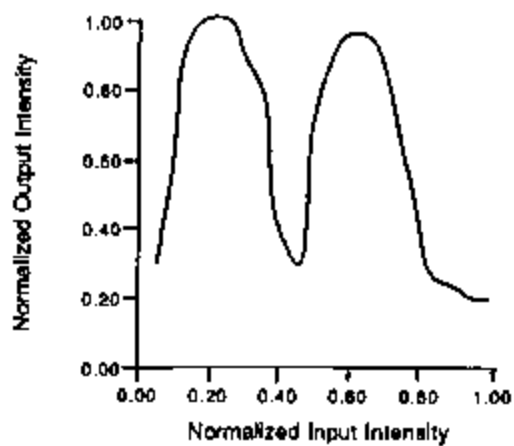


Figure 1 A coherent-optic image-processing system.



2(a)



2(b)

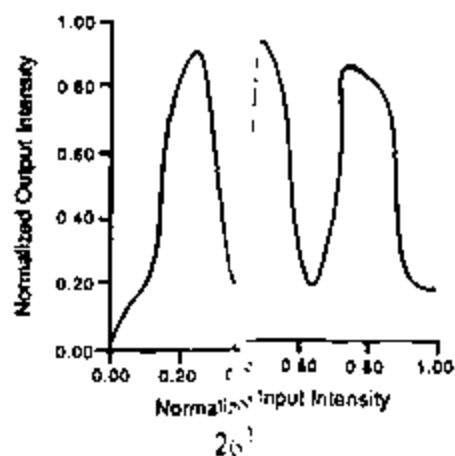


Figure 2 Experimental simulation of nonlinear optical processing. An example of nonmonotonic transformation for (a) the first, (b) second, and (c) third orders.

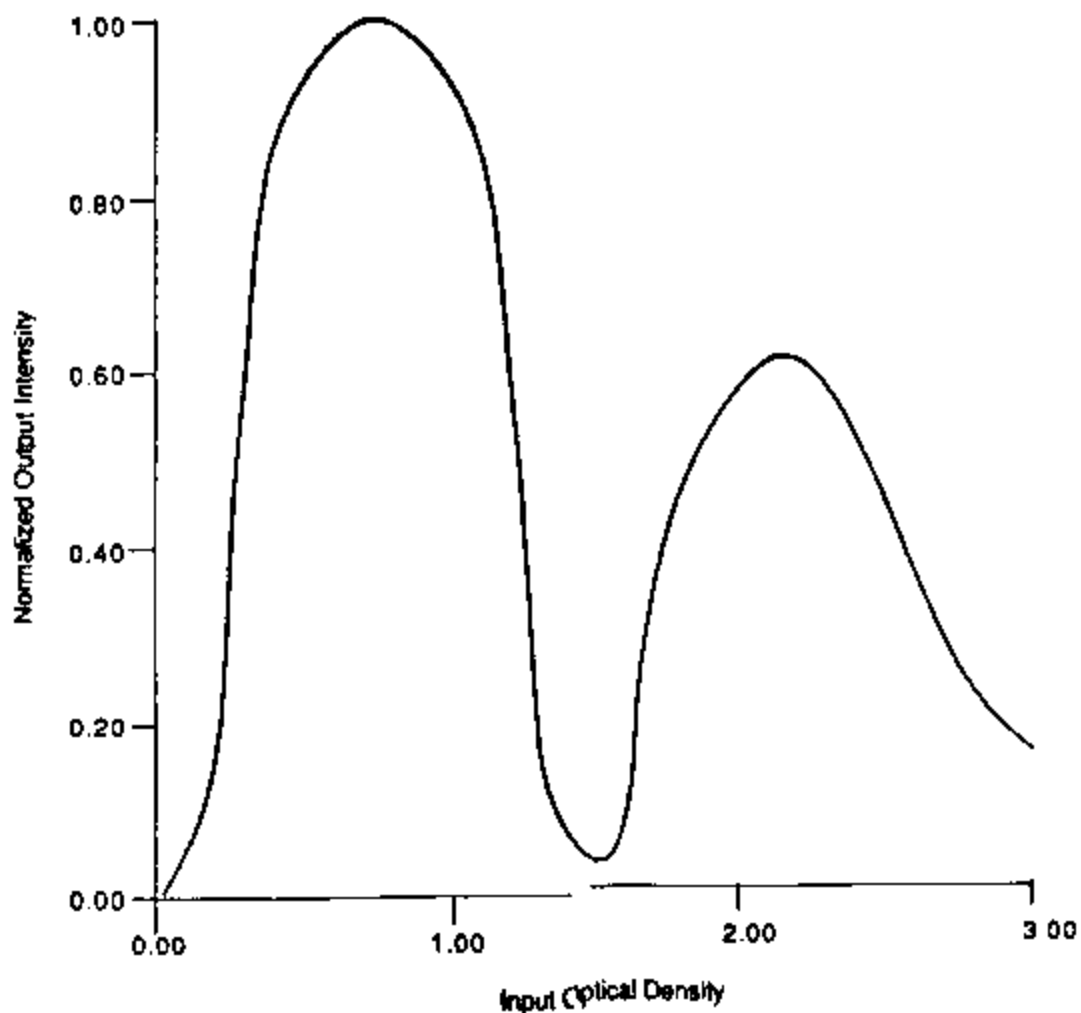


Figure 3-5 An example of nonmonotonic transformation achieved with phase coding. The first diffraction order is selected.

This type of grating cannot be considered to be a "thin" grating and its diffraction efficiency as a function of groove depth is described by the rigorous coupled wave theory. The resulting function is more like a sinc-squared form. We examined photoresist as a recording material for nonlinear processors by copying a Ronchi type grating on it, so that a square-wave surface relief grating is recorded. The transfer function for this case is shown in Figure 4. Although the dynamic range is narrower than that of DCG, the maximum achievable modulation is higher, several peaks occur in the transfer function. This is a useful transformation for a equidensitometer.

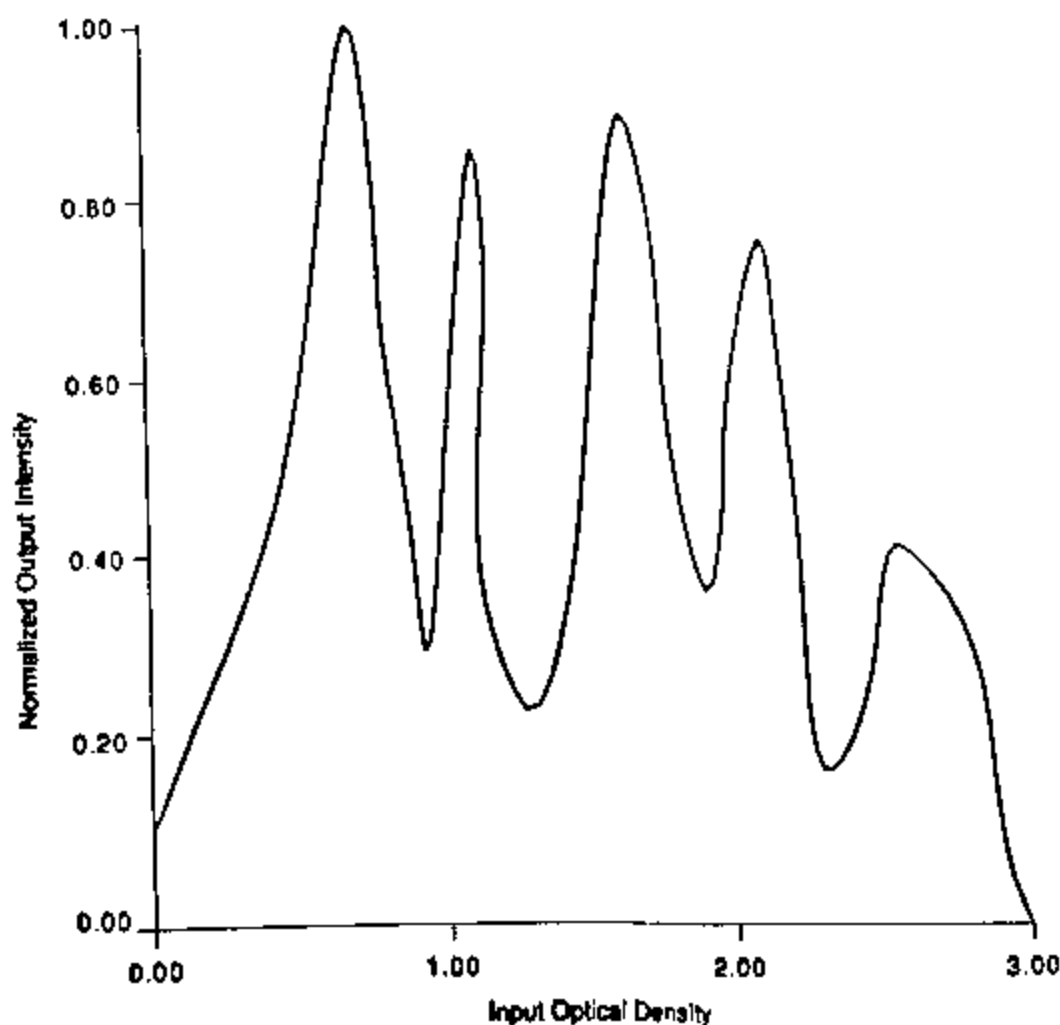


Figure 4 Multippeak transfer function of optical processor using photoresist as a phase recording material.

The comparison of two types of nonlinear optical processors is given in Table 2.

Table 2

NONLINEAR OPTICAL PROCESSING IN A LINEAR OPTICAL SYSTEM	
Intensity-to-space modulation (Halftoning)	Intensity-to-phase modulation
low light efficiency	high light efficiency (high diffraction orders can be used)
"hard-clipping" recording media	phase recording media
special halftone screen design or filtered diffraction pattern	simple types of gratings (sinusoidal, rectangular, etc.)
multipeak transformation in high diffraction orders only	multipeak transformation in the first order
real-time implementation using OASLM	real-time implementation using OASLM or erasable photopolymers
input dynamic range 30:1	input dynamic range (experimentally proven) 1000:1

#### 4. ACKNOWLEDGEMENT

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## 5. REFERENCES

1. F. Talbot, "Facts Relating to Optical Science No. IV," *Philos. Mag* 9, 401-407 (1836).
2. John W. Wesner in "Screen Pattern Used in Reproduction of Continuous-Tone Graphics," *Appl. Opt.*, 13 (7), 1703-1710 (1974) Cites H.A. Groesbeek, Jr., *The Process and Practice of Photo-Engraving* (Doubleday, Garden City, N.Y., Page & Co., (1024) as a source of this information.
3. M. Marquet and H. Tsujiuchi "Interpretation des Aspects Particuliers des Images Obtenues dans une Experience de Detramage," *Opt. Acta* 8, 267-277 (1961).
4. H. Kato and J. W. Goodman, "Nonlinear Transformations and Logarithmic filtering in Coherent Optical Systems," *Opt. Comm.*, 8 (4), 378-381 (1973).
5. S.R. Dashiell and A.A. Sawchuck, "Nonlinear Optical Processing: Analysis and Synthesis," *Appl. Opt.*, 16 (4), 1009-1025 (1977).
6. S.R. Dashiell and A.A. Sawchuck, "Nonlinear Optical Processing: Effect of Input Medium and Precompensation," *Appl. Opt.* 16 (8), 2279-2287 (1977).
7. H.K. Liu, J.W. Goodman, and J.L.H. Chan "Equidensitometry by Coherent Optical Filtering," *Appl. Opt.*, 15 (10), 2294-2399 (1976).
8. S. Matsumoto and B. Liu "Halftone Screen Design Methods for Nonlinear Optical Processing," *Appl. Opt.*, 18 (16), 2792-2802 (1979).
9. T.C. Strand "Non-Monotonic Nonlinear Image Processing Using Halftone Techniques," *Opt. Comm.*, 15 (1), 60-65 (1975).
10. A. Armand, A.A. Sawchuck, and T.C. Strand "Nonlinear Optical Processing with Halftones: Accurate Predictions for Degradation and Compensation".
11. T.K. Gaylord, M.G. Moharam "Analysis and Application of Optical Diffraction by Gratings" *IEEE Proc.*, 73, 894 (1985).